**F distribution**

- It is **positively skewed** or the tail goes indefinitely into the **positive** direction because values are always **positive**. This also means that we could never get a negative F ratio.
- A family of distributions dependent on **df**
- Looks like we are performing a one-tailed test and we are in terms of F distribution but in terms of hypothesis testing, we are still doing a **non-directional test**
- We round down the F critical to be more conservative

***<u>Steps in doing an one-way between-subjects/independent groups ANOVA</u>***

*<u>Step 1:</u>* **compute the grand mean or the mean score of everyone in the study**

*Two ways of doing this*

a. **Adding all scores up and divide by the total number of participants**
b. **Compute the mean of each condition and add them up and divide by the total number of conditions**

*<u>Step 2:</u>* **calculate SStotal =** $\sum(\mathbf{x_i} - \bar{\mathbf{x}}_{\mathbf{grand}})^2$ **= ∑ individual score - grand mean**

*<u>Step 3:</u>* **calculate the SStreat =** $\sum \mathbf{n_x}(\bar{\mathbf{x}}_{\mathbf{k}} - \bar{\mathbf{x}}_{\mathbf{grand}})^2$

*<u>Step 4:</u>* **calculate the SSerror =** $\sum(\mathbf{x_{ik}} - \bar{\mathbf{x}}_{\mathbf{k}})^2$ **= ∑ individual score in each condition - mean of each condition**

Or **SStotal = SStreatment + SSerror**

*<u>Step 5:</u>* **calculate the three degrees of freedom**

*<u>Step 6:</u>* **calculate mean square treatment =** $\mathbf{MStreat} = \frac{\mathbf{SStreat}}{\mathbf{dftreat}}$ **and error**

$$\mathbf{MSerror} = \frac{\mathbf{SSerror}}{\mathbf{dferror}}$$

*Step 7:* **calculate F ratio = F =** $\frac{\mathbf{MStreat}}{\mathbf{MSerror}}$

*Step 8:* **$F_{.05}$ = (df for treatment, df for error)**

Ex: An experiment was conducted to determine the effect of restricting certain types of food on positive mood. Thirty participants were randomly assigned to one of three diets (restrict fat, restrict carbohydrates, no restrictions), with ten people following each diet. Participants' mood ratings, on a scale from 1 (wading through the sea of despair) to 16 (radiantly happy), were assessed

| *Diet 1: restrict fat* | *Diet 2: restrict carbs* | *Diet 3: no restrictions* |
|---|---|---|
| 7 | 10 | 12 |
| 9 | 13 | 11 |
| 8 | 9 | 15 |
| 12 | 11 | 7 |
| 8 | 5 | 14 |
| 7 | 9 | 10 |
| 4 | 8 | 12 |
| 10 | 10 | 12 |
| 9 | 8 | 13 |
| 6 | 7 | 14 |
| $\overline{X}_k$ 8 | 9 | 12 |

H0: mood doesn't vary as a function of diet

$\mu 1 = \mu 2 = \mu 3$ or $\mu_{\text{restric carbs}} = \mu_{\text{restrict fat}} = \mu_{\text{no restrictions}}$

H1: mood varies as a function of type of diet

$\mu k \neq \mu k'$

*Step 1:*

Grand mean = (8 + 9 + 12)/3 = **9.667**

- Irrespective of the diet, the average happiness of these 30 participants is 9.667

*Step 2:*

$SS_{Total} \quad \sum(x_i - x_{grand})^2 \quad \sum(x - x_.)^2 \quad \overline{X}_. = 9.667$

| $X_1$ | $X_1 - \bar{X}$ | $(X_1 - \bar{X})^2$ | $X_2$ | $X_2 - \bar{X}$ | $(X_2 - \bar{X})^2$ | $X_3$ | $X_3 - \bar{X}$ | $(X_3 - \bar{X})^2$ |
|---|---|---|---|---|---|---|---|---|
| 7 | -2.667 | 7.112 | 10 | 0.333 | 0.111 | 12 | 2.333 | 5.443 |
| 9 | -0.667 | 0.445 | 13 | 3.333 | 11.109 | 11 | 1.333 | 1.777 |
| 8 | -1.667 | 2.78 | 9 | -0.667 | 0.445 | 15 | 5.333 | 28.441 |
| 12 | 2.333 | 5.44 | 11 | 1.333 | 1.777 | 7 | -2.667 | 7.113 |
| 8 | -1.667 | 2.78 | 5 | -4.667 | 21.781 | 14 | 4.333 | 18.775 |
| 7 | -2.667 | 7.11 | 9 | -0.667 | 0.445 | 10 | 0.333 | 0.111 |
| 4 | -5.667 | 32.115 | 8 | -1.667 | 2.779 | 12 | 2.333 | 5.443 |
| 10 | 0.333 | 0.111 | 10 | 0.333 | 0.111 | 12 | 2.333 | 5.443 |
| 9 | -0.667 | 0.445 | 8 | -1.667 | 2.779 | 13 | 3.333 | 11.109 |
| 6 | -3.667 | 13.447 | 7 | -2.667 | 7.113 | 14 | 4.333 | 18.775 |
| Σ | | 71.789 | | | 48.449 | | | 102.429 |

71.789 + 48.449 +102.429 = SStotal = **222.667**

*Step 3:*

$$SStreat = 10(8-9.667)^2 + 10(9-9.667)^2 + 10(12-9.667)^2$$
$$= 10(-1.667)^2 + 10(-0.667)^2 + 10(2.333)^2$$
$$= 27.789 + 4.449 + 54.429$$
$$= 86.667$$

*Step 4:*

| $X_1$ | $X_{i1} - \bar{X}_1$ | $(X_{i1} - \bar{X}_1)^2$ | $X_2$ | $X_{i2} - \bar{X}_2$ | $(X_{i2} - \bar{X}_2)^2$ | $X_3$ | $X_{i3} - \bar{X}_3$ | $(X_{i3} - \bar{X}_3)^2$ |
|---|---|---|---|---|---|---|---|---|
| 7 | -1 | 1 | 10 | 1 | 1 | 12 | 0 | 0 |
| 9 | 1 | 1 | 13 | 4 | 16 | 11 | -1 | 1 |
| 8 | 0 | 0 | 9 | 0 | 0 | 15 | 3 | 9 |
| 12 | 4 | 16 | 11 | 2 | 4 | 7 | -5 | 25 |
| 8 | 0 | 0 | 5 | -4 | 16 | 14 | 2 | 4 |
| 7 | -1 | 1 | 9 | 0 | 0 | 10 | -2 | 4 |
| 4 | -4 | 16 | 8 | -1 | 1 | 12 | 0 | 0 |
| 10 | 2 | 4 | 10 | 1 | 1 | 12 | 0 | 0 |
| 9 | 1 | 1 | 8 | -1 | 1 | 13 | 1 | 1 |
| 6 | -2 | 4 | 7 | -2 | 4 | 14 | 2 | 4 |
| $\bar{X}_1$ = 8 | | 44 | $\bar{X}_2$ = 9 | | 44 | $\bar{X}_3$ = 12 | | 48 |

$SS_{error}$ = 44 + 44+ 48 =136

Or SSerror = 222.667 - 86.667 = **136**

*Step 5:*

dftotal = 30 - 1 = **29**

dftreatment = 3 - 1 = **2**

dferror = (10 - 1) + (10 - 1) + (10 - 1) = **27**

*Step 6:*

$$\text{MStreat} = \frac{\text{SStreat}}{\text{dftreat}} = \frac{86.667}{2} = \mathbf{43.333}$$

$$\text{MSerror} = \frac{\text{SSerror}}{\text{dferror}} = \frac{136}{27} = \mathbf{5.037}$$

*Step 7:*

F = $\frac{\text{MStreat}}{\text{MSerror}} = \frac{43.333}{5.037} = 8.603$

***Summary table***

| Source | SS | df | MS | F |
|---|---|---|---|---|
| Treatment | 86.667 | 2 | 43.333 | 8.603 |
| Error | 136 | 27 | 5.037 | |
| Total | 222.667 | 29 | | |

*Step 8:*

$F_{0.05}$ (2,27) = **3.37**

*Step 9:*

Reject the null hypothesis. A one-way between-subjects ANOVA revealed that mood varied significantly with type of diet, $F(2,27) = 8.60$, $p < 0.05$

- We can't say which diet could produce the highest happiness score or the lowest happiness score, all we can say is that mood differs with the type of diet