

ENG1001 Engineering Design 1

Structure & Loads

Determine forces that act on structures causing it to deform, bend, and stretch
Forces push/pull on objects

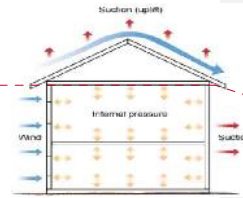
Structures are **loaded** by:

- > Dead loads – permanent fixed loads (all fixed attachments i.e partitions, tiles etc)
- > Live loads – Temporary or transient loads (Removable loads i.e people, furniture)

Dead and live loads are **vertical forces** due to gravity (gravity gives weight to objects). Objects experience **force** of gravity directed to centre of earth

- > Wind loads – Act in any direction, act inside and out, are a function of:

Building size and **shape**, height off ground, geographical location, surrounding obstructions



Commented [tr1]: Earthquake loads
Earth pressure loads (basement walls)
Liquid loads (bridge piers)
Thermal loads

Commented [tr2]:

Force (or weight) = mass*gravity
Units: N (newtons) = kg*m/s²

Commented [tr3]: More streamlined the shape, the less the force





- Load Path

Route along which applied loads are transferred to support (ground)

Vary as function of structure (bird on roof, weight on roof sheet to purlins to truss to ground)

- Load transfer

Process of load transfer involves structural actions

	Process	Structural actions	Deformation
1		Axial compression or tension	squashing or stretching
2		bending	curving
3		shear	distortion
4		torque	twisting

One face squashes, other stretches

Commented [tr4]: Moments cause bending, curving
Torque causes twisting

Structural Forms	Structural Actions	Load transfer
Arches (domes)	Compression	Axial compression – Vertical loads are turned into radial compression forces which are then reacted by the abutments
Beams (slabs)	Bending & shear	Horizontal member supporting vertical loads; materials that transfer both tensile and compressive stresses (timber, steel) Beams transfer load by bending
Cables (membranes)	Tension	Axial tensions
Trusses (space frame)	Tension or compression	Axial tension & compression Truss transfer load by members being in either tension or compression Triangulated structures (efficient for large spans; top/bottom chords, diagonals and vertices)

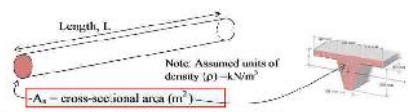
- **Calculating loads** dead (self weight) and live loads

Water = 1000 kg/m^3 (10 kN/m^3)
 Concrete = 2400 kg/m^3 (24 kN/m^3)
 Steel = 7850 kg/m^3 (78.5 kN/m^3)

Self-Weight (kN) = $W_{\text{total}}(\text{kN}) = \text{Volume} * \text{density}$

Self-Weight (kN/m) = $W(\text{kN/m}) = A_x * \text{density}$

- **Self weight of a member**



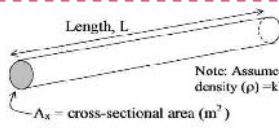
Note: Assumed units of density (ρ) = kN/m^3

A_x = cross-sectional area (m^2)

$W_{\text{total}}(\text{kN}) = \text{Volume} * \text{density}$

$$w(\text{kN/m}) = \frac{\text{Volume} * \text{density}}{\text{Length}} = \frac{(A_x * L) * \text{density}}{L}$$

$= A_x * \text{density}$



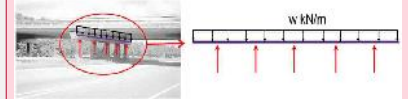
Note: Assumed units of density (ρ) = kN/m^3

A_x = cross-sectional area (m^2)

Check units: $w(\text{kN/m}) = A_x * \text{density}$

$$\frac{[\text{kN}]}{[\text{m}]} = [\text{m}^2] * \frac{[\text{kN}]}{[\text{m}^3]} = \text{kN/m} \Rightarrow \text{OK}$$

Commented [tr5]: use line models



Commented [tr6]:

Self weight of a Slab (kN/m^2)

NOTE: units of density (ρ) = kN/m^3

Length, L

Breadth, B

thickness, t (m)

$W_{\text{total}}(\text{kN}) = \text{Volume} * \text{density}$

$$w_s(\text{kN/m}^2) = \frac{\text{Volume} * \text{density}}{\text{Plan Area}} = \frac{(B * L * t) * \text{density}}{(B * L)}$$

$= t * \text{density}$

Example

NOTE: units of density (ρ) = kN/m^3

Length, L

Breadth, B

thickness, t (m)

Q. A 150mm thick concrete slab is 4m by 5m.
 How much does it weigh in kN and KN/m^2 ?

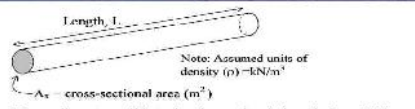
Volume = $L * B * t = 4 * 5 * 0.15 = 3 \text{ m}^3$; $\rho_{\text{conc}} = 24 \text{ kN/m}^3$

$w(\text{kN}) = \text{density} * \text{vol} = 24 * 3 = 72 \text{ kN}$

$t = 0.15 \text{ m}$; $\rho_{\text{concrete}} = 24 \text{ kN/m}^3$

$w(\text{kN/m}^2) = \text{density} * t = 24 * 0.15 = 3.6 \text{ kN/m}^2$

Example 1



Note: Assumed units of density (ρ) = kN/m^3


A_x = cross-sectional area (m^2)

Q. We have 50mm diameter solid steel rods varying in length from 5-10m.
 How much do they weigh (kN/m)? $\rho_{\text{steel}} = 78.5 \text{ kN/m}^3$

$A_x = \pi r^2 = \pi (0.025)^2 = 0.00196 \text{ m}^2$ (Change units to m^2)

$w(\text{kN/m}) = \rho * A_x = 78.5 * 0.00196 = 0.154 \text{ kN/m}$

Example 2



Pre-cast beam cross-section of CUBE6 flooring system

Q. A concrete precast floor beam is shown above. It's length will vary depending on its design location. How much does the beam weigh in kN/m ?

$A_x(\text{cross sectional area}) = 42 * 100 + 50 * 92 = 8800 \text{ mm}^2$

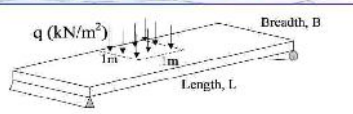
$\rho_{\text{conc}} = 24 \text{ kN/m}^3$

$w(\text{kN/m}) = \rho * A_x = 24 * (8800 * 10^{-6}) = 0.211 \text{ kN/m}$ ($\approx 21 \text{ kg/m}$) (Change units to m^2)

- **Calculating Live loads**

kPa = kN/m^2 e.g. $3 \text{ kPa} = 3 * 100 \text{ kg}$ people standing in every m^2

Converting Live load from kN/m^2 to kN/m



$q(\text{kN/m}^2)$

Breadth, B

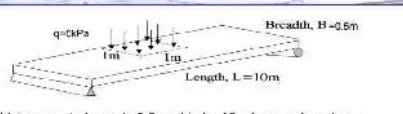
Length, L

$LL_{\text{total}}(\text{kN}) = q * (\text{Plan Area}) = q * (B * L)$

$$LL(\text{kN/m}) = \frac{q * (\text{Plan Area})}{\text{Length}} = \frac{q * (B * L)}{L}$$

$= q * B$

Example



$q = 5 \text{ kPa}$

Breadth, B = 0.6m

Length, L = 10m

Q. A 150mm thick concrete beam is 0.6m wide by 10m long and carries a live load = 5 kPa (kN/m^2). It is supported at its ends, 10m apart. What is the live load on the beam in kN/m ?

$q(\text{kN/m}) = B * q = 0.6 * 5 = 3.0 \text{ kN/m}$

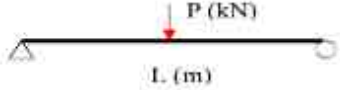
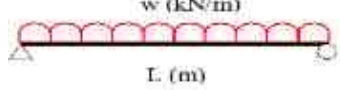

- **Converting units**

E.g. Water = 1000 kg/m^3 (10 kN/m^3)
 Show in g/cm^3

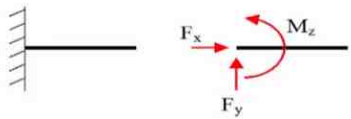
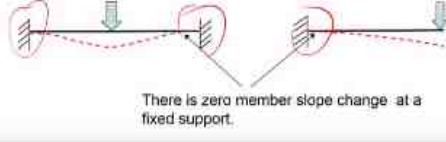
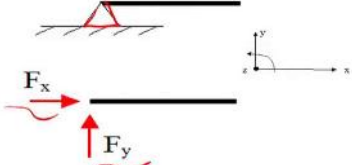

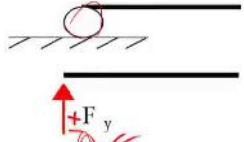
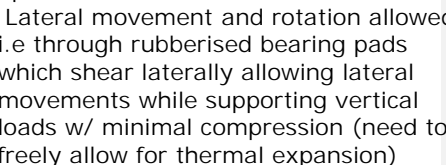
$$\Rightarrow 1000 \frac{\text{kg}}{\text{m}^3} = 1000 \frac{(1000 \text{ g})}{(100 \text{ cm})^3} = \frac{10^6}{10^6} \frac{\text{g}}{\text{cm}^3} = 1 \text{ g/cm}^3$$

1000g = 1kg 100cm = 1m

Calculating Reactions

Idealised Loads		
Type of load	Simple Structural Model	
Point Loads		Single arrow at concentrated force
Uniformly Distributed (line) Loads		Load per m length of beam (kN/m) In 3D: area load (kN/m ²) kPa acting on plan area (
Applied Loads/moment		Result of two applied forces not being in line Moment=Force*d (Nm)

Commented [tr9]: Common units of Load
 kN – concentrated/point load
 kN/m – line load
 kN/m² – area load (stress)
 kNm – applied moment load

Idealised Supports		
Type of support	Simple Support Model	Deflected shape
Fixed support	 Fixed support provides reactions to prevent translation (forces) & rotation. Force and moment reactions occur	 There is zero member slope change at a fixed support.
Simple/Pin support	 Member cannot translate at support, but can rotate. Only force reactions occur	 As long as <u>some</u> rotation is possible, the support is considered a pin/simple support
Roller support	 Structure is free to move in one direction & rotate. Support provides one force reaction only to prevent translation in one direction.	 Single force reaction is ± restraint. Weight of the structure prevents upwards movement. Lateral movement and rotation allowed i.e through rubberised bearing pads which shear laterally allowing lateral movements while supporting vertical loads w/ minimal compression (need to freely allow for thermal expansion)

Trusses

- How does a Truss work?

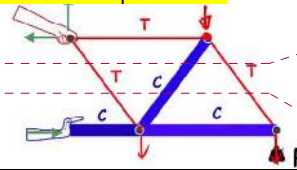
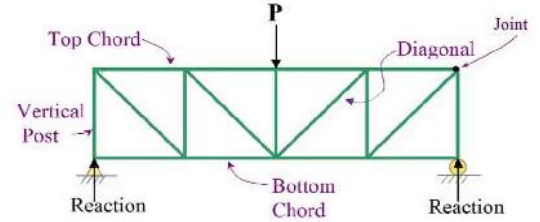
- Remember structures carry loads in four ways via structural actions
- > Axial compression or tension (stretching/shortening)
 - > Bending (curving)
 - > Shear (distortion)
 - > Torque (twisting)

Individual members in a truss experience either axial tension or compression

Here, two mathematical assumptions are being made:



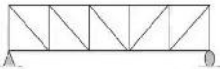

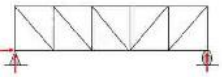
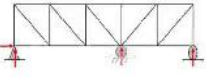

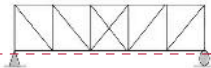
- 1) Trusses are loaded and supported only at joints
- 2) All joints in a truss are pin joints

- Stability & Determinacy of Trusses



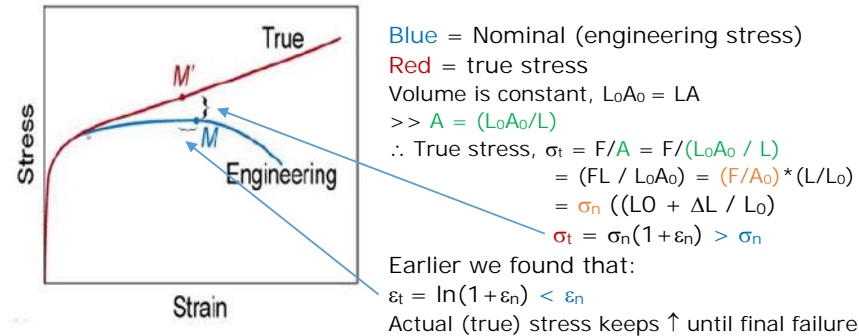
Commented [tr12]: Applied loads at stringers, load transfers through stringer to secondary beam member and then appears on truss at joint location

Commented [tr13]: In practice truss joints are usually semi-rigid, if loaded at joints; moments & shear forces are small compared to internal axial forces \therefore simplifying assumption of pin joints is acceptable

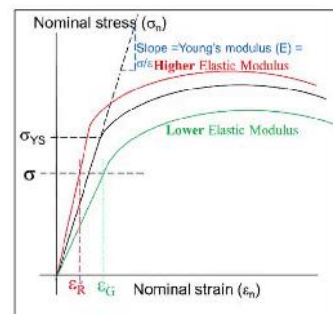
External & Internal Stability of Trusses			
Convention	Stable Model	Unstable Model	
Externally Stable Supports provide adequate rotational & translational restraint	 <p>Stable ✓</p> <p>3 restraints ensure no translation in x and y direction and rotation</p>	 <p>Unstable ✗ (2 roller supports)</p> <p>No horizontal restraint</p>	
Internally Stable Arrangement of members can carry the applied loads through to support	 <p>Fully triangulated ∴ stable</p>	 <p>Unstable ✗ (not triangulated)</p> <p>Not fully triangulated, assume pin joints if one applied vertical load we see that movement</p>	
External & Internal Determinacy of Trusses			
Externally Determinate Supports provide just enough restraint i.e. reactions calculator using equilb.	 <p>Externally Determinate (3 unknowns)</p>	Externally Indeterminate Supports provide extra restraint	 <p>Externally Indeterminate (4 unknowns)</p>
Internally Determinate Member forces can be determined using equilb.	 <p>Internally Determinate</p>	Internally Indeterminate Truss has extra members, so equilb doesn't provide enough eq ^{ns}	 <p>Internally Indeterminate</p>

Commented [tr14]: Indeterminate trusses have more than one load path

- Measuring mechanical properties of materials



- What properties are we interested in



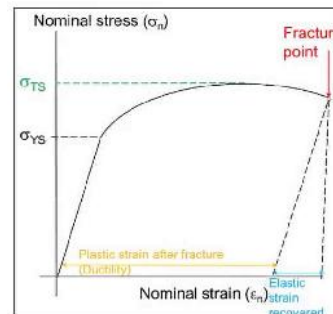
σ_{YS} = Yield stress (Yield strength)
 Once plastically deformed, dimensions changed permanently
 $\epsilon = E \epsilon$ (Hookes Law; valid for the linear Elastic region)
 $E (= \sigma/\epsilon)$ – Young's Modulus (Elastic modulus)

If we stress two materials to same point, higher modulus material will elongate a certain amount, lower modulus material at the same stress will elongate more

Commented [tr23]: Criteria for failure

Commented [tr24]: We can determine Young's modulus (E) by initial slope; $E = \sigma/\epsilon$

This impacts stiffness of material; important in deflection limit design, want material to withstand certain force and only deflect certain amount

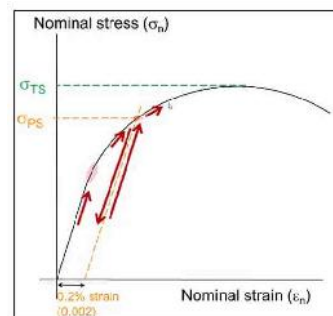


σ_{TS} = Tensile strength (Ultimate Tensile Strength – UTS); Occurs at peak

Measuring a specimen after its broken, it doesn't have same strain, it has a smaller strain. Elastic strain has been recovered (as its fully recoverable) which we get back after fracture, where then left w/ plastic strain after fracture (ductility)

\therefore Elongation at point of fracture = Ductility + Elastic strain

Commented [tr25]: Ductility only looks at plastic strain, we recover some strain from elastic component



σ_{PS} = Proof Stress (offset yield strength) ($\sigma_{0.2\%}$ strain)

E.g.

If we took sample to 0.2% strain then release and re-apply the load, we go past initial yield strength and it stays linear (\uparrow elastic portion of material; yield strength \uparrow , and once we get past this new yield strength we will plastically deform).

Commented [tr26]: This is called work hardening; we strengthen material by plastically deforming it

- Buckling

Members under axial compression are usually weaker than that under axial tensions because of buckling (which equates to a member failing)

Load at which member buckles or fails is defined by Euler buckling formula


Euler Buckling Load


$$P_e = \frac{\pi^2 EI}{(L_e)^2}$$


E = Young's modulus
 I = weakest 2nd moment of area or moment of inertia
 L_e = Effective length


Second moment of area, I

Describes bending/buckling stiffness of a cross-section

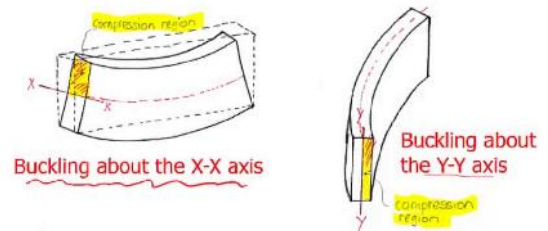
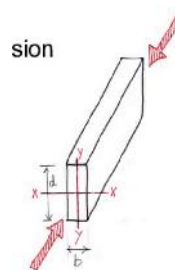
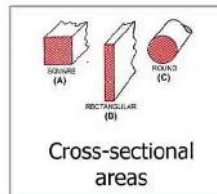

 $I = \frac{b^4}{12}$


 $I = \frac{b_o^4}{12} - \frac{b_i^4}{12}$


 $I = \frac{\pi r^4}{4}$


 $I = \frac{\pi(r_o^4 - r_i^4)}{4}$

I values for square and circular cross-sectional areas



A section has 2 possible axes which it might buckle (x and y axis, 90° apart)

Buckle axes divides region in compression from region in tension

>> A section will always buckle about its least stiff (weaker) axis

- Eulers Buckling

E , I and π are constant \therefore

$$P_e = \frac{\pi^2 EI}{(L_e)^2} \quad P_e \propto \frac{1}{(L_e)^2}$$

E.g.

Example

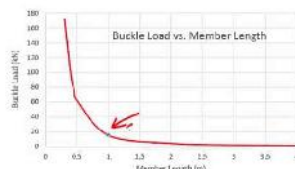
Q. What is the compressive buckle capacity of a 1m long 10mm radius solid steel rod.
Assume $E=200,000\text{MPa}$

$$P_e = \frac{\pi^2 EI}{(L_e)^2}$$

$$I = \frac{\pi r^4}{4} = \frac{\pi (10)^4}{4} = 7854 \text{ mm}^4$$

$$P_e = \frac{\pi^2 \times 200,000 \times 7854}{(1000)^2} = 15,500\text{N} \text{ or } 15.5\text{kN}$$

Assume both supports to be pin supports



Commented [tr32]: UNITS:
Use N, mm and MPa (N/mm²)

$$[N] = \frac{[N / \text{mm}^2][\text{mm}^4]}{[\text{mm}^2]}$$

Kinetics

How forces influence acceleration

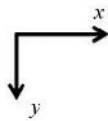
- Newtons Laws of Motion

1. A particle at rest or moving in a straight line w/ constant velocity, will remain in this state provided the particle is not subjected to an unbalanced force
2. A particle acted upon by an unbalanced force (F) experiences an acceleration (a) that has the same direction as the force and a magnitude that is directly proportional to the force
 $F = m \cdot a$
3. Mutual forces of actions and reaction between two particles are =, opposite and collinear

- FBDs and Dynamics

FBD shows all forces acting on body

1. Define coordinate system
2. Draw in forces you want to include



- Forces to Consider

Common forces you'd expect to encounter

It's vital we include all relevant forces acting on all masses in our system

Gravity [mg]

$F = m \cdot g$ at sea level
 $g = 10 \text{ m/s}^2$

Normal Force [F_n]

Force between two objects contacting

Normal force between surface of mass and surface its resting on

Always outward from surface



Friction

Acts in opposite direction to movement

$F = u \cdot F_n$

u = coefficient of friction (material dependant)

F_n = normal force

Elastic forces

$F = k \cdot x$

k = spring coefficient

x = displacement (direction of displacement is important)

Damping

When you move through a fluid it provides damping

Damping acts against direction of motion; linear damping is modelled by:

$F = c \cdot v$

c = damping coefficient

v = velocity (damping is dependent on velocity)

Particle Rotation

When a body rotates about a fixed axis it experiences two accelerations:

Radial acceleration – proportional to angular velocity
accelerating towards centre

Tangential acceleration – proportional to the angular acceleration
accelerating in direction of motion

- Fundamental definitions

Set up rotation coordinate system: CCW = positive

Angular Position:

SI units: radians

$$\theta = \frac{\text{Arc Length}}{\text{Radius}}$$

Angular Displacement:

*Can be vectors

$$\Delta\theta = \theta_2 - \theta_1$$

Angular Velocity:

SI units: radians/second

$$\omega_{ave} = \frac{\Delta\theta}{\Delta t} = \frac{(\theta_2 - \theta_1)}{(t_2 - t_1)}$$
$$\omega = \frac{d\theta}{dt}$$

Angular Acceleration:

SI units: radians/second²

$$\alpha_{ave} = \frac{\Delta\omega}{\Delta t} = \frac{(\omega_2 - \omega_1)}{(t_2 - t_1)}$$
$$\alpha = \frac{d\omega}{dt}$$

