FINS2624 Summary

1- Bond Pricing

Yield to Maturity: The YTM is a hypothetical and constant interest rate which makes the PV of bond payments equal to its price; considered an average rate of return. It implies a flat term structure of interest rates but does not require a flat term structure of interest rates for it to correctly price bonds.

Realised Compound Yield: It is the investment return at the maturity of the bond given that the annual coupons were reinvested. This is thought of as an outcome as it is realised, and is generally used for bonds. It is similar to the **Holding Period Return**, but this can be realised or stochastic and refers to any investment

- $r_A = (1 + r_s)^{\frac{1}{T}} - 1$ are the annualised returns
- r_s are the simple returns or change in wealth ((P_{new} - P_{old}) / P_{old})

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2 - The Term Structure of Interest Rates

T Spot Rate: The fixed interest rate for an investment starting today and ending at time *t*. It can be denoted as Y_t / ${}_0Y_t$ when starting at *t=0*, or ${}_1Y_2$ to denote a starting and end time

Term Structure of Interest Rates: It is the pure yield curve made up of the various *t spot rates*, plotting the YTM as a function of the time to maturity. We can use these to price bonds if they are appropriate to its maturity.

- Through a process of **bootstrapping**, we can *back out* the spot rates of bonds to find the implied price.

Arbitrage Opportunities: Arbitrage opportunities rely on creating synthetic instruments which allow us to exploit mispricings to create an arbitrage trade. To do so:

1. Find the arbitrage free/implied price of a bond through the process of bootstrapping to identify arbitrage opportunities
2. Set up an arbitrage trade
3. Construct a synthetic bond; essentially constructing scales versions of bonds with the same or shorter maturity and combining them to replicate the cash flows needed
4. Exploit mispricing with a long and short position

Forward Rates: Forwards rates that interest rates for investments we agree on today but take place in the future. It can be denoted as ${}_1f_2$, which is the fwd. rate valid at *t=1* for reinvestment between *t=1* & *t=2*.

- To find forward rates, set up a replicating strategy

Liquidity Risk vs. Reinvestment Risk: When our investment horizon (preferred habitat) does not match cash flows, risk arises as we do not know the forward rates, price and subsequently the returns.

- *Liquidity Risk* - investment horizon is less than cash flows
- *Reinvestment Risk* - investment horizon is greater than cash flows

Liquidity Preference/Preferred Habitat Theory: Suggests that the shape of the term structure is determined by the relative horizons of issuers and investors:

- *Issuer horizon > investor horizon:* yield curve is upward sloping as issuers (borrowing money) will need to offer higher yields to compensate investors for additional liquidity risk; known as the liquidity premium
- *Investor horizon > issuer horizon:* yield curve is downward sloping as investors will pay higher prices and lower yields so that issuers are compensated for additional reinvestment risk

Expectations Hypothesis: Theory that market expectations of future interest rates shape the term structure s.t. all forward rates equal the expectations of future spot rates

$${}_sf_t = E\left({}_sy_t\right)$$

Combining EH & Preferred Habitat Theory:

$${}_sf_t = E\left({}_sy_t\right) + L$$

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3 - Duration

Duration *(D)*: Duration is the average time you have to wait for a present value dollar (expressed in years), and can be thought of as an adjusted time to maturity

- It is calculated as the weighted average of maturity with weights being the ratio of the PV of cash flows at each time to the price

FINS2624 Notes

1 - Bond Pricing

- Bond prices which are quoted are the "clean price," excluding accrued interest. However, the bond's market value is actually its "dirty price," with accrued interest being paid.
  - Dirty price = clean price + accrued interest
- **Assumptions**:
  - No default risk
  - No transaction costs
  - Constant interest rates
  - Complete markets
- **Approaches to pricing** - fundamental vs. arbitrage
  - Fundamental pricing - based upon supply-demand equilibrium & can be used to price stocks
  - Arbitrage pricing - assume that current market prices are in equilibrium and set prices relative to these using arbitrage-free prices & can be used to price bonds and derivatives
    - **Arbitrage** are trades that generate zero cash flows in the future but a positive and risk free cash flow today
      - These types of trades violate the principle of the Law of One Price
      - If there are arbitrage opportunities, we can only say that the prices are **internally inconsistent**, but not that the bond price or bank interest rate were wrong
    - **The price of a financial asset is the sum of the present values of its future cash flows**
- **Replicating portfolios** - constructing replicating portfolios using assets with known prices to mimic the cash flows of other assets
  - Example: replicating bonds by "putting money into a bank" based on the no arbitrage argument
  - Discounting to find NPV is a short method of using this concept, based on the Law of One Price
  - Note: interest rate may vary according to the length of a bank deposit
- **Annuity** - formula for T periods can be derived by considering the difference between two perpetuities:
  - One starting at T=1
  - One starting at T=T+1

$$PV(CS) = \frac{c}{y} - \frac{c}{y}\Big/(1+y)^T = \frac{c}{y}\left[1 - \frac{1}{(1+y)^T}\right]$$

- The **YTM** is a hypothetical and constant interest rate which can be used to value a bond - it is the interest rate which makes the PV of bond payments equal to its price and is considered an average rate of return; WHILE THE YTM IMPLIES A FLAT TERM STRUCTURE OF INTEREST RATES, DERIVING THE YTM DOES NOT REQUIRE A FLAT TERM STRUCTURE OF INTERESTS FOR IT TO CORRECTLY

PRICE A BOND

- **There is an inverse relationship between the yield/interest rate and the bond price** - however, due to **convexity,** the price is less sensitive to changes in the YTM when the YTM is high
- Note:
  - Increased Coupon rate (C) = Increased Price (P)
  - Increased Yield (YTM) = Decreased Price
- Bond trades at
  - par --> P = FV (C = YTM)
  - discount --> P < FV (C < YTM)
  - premium --> P > FV (C > YTM)

- The **realised compound yield** is the investment return at the maturity of the bond, given that annual coupons were reinvested; This is the essentially the same as the Holding Period Return BUT THE REALISED COMPOUND YIELD IS GENERALLY THOUGHT OF AS AN OUTCOME AS IT IS REALISED; MORE SO FOR BONDS. THE HPR CAN BE REALISED OR STOCHASTIC AND REFERS TO ANY INVESTMENT
  - $r_A = (1 + r_s)^{\frac{1}{T}} - 1$; annualised returns
  - Rs is the simple returns which is just the change in wealth ((Pnew-Pold)/Pold)
- Other Notes:
  - Alternative YTM interpretation is not used - concept that if all coupons are reinvested at the YTM rate, the realised compound yield would be equal to the YTM
  - **Current yield = annual coupon/price**

2 - The Term Structure of Interest Rates

- The **t spot rate** is the fixed interest rate for an investment starting today and ending at time "t"
  - Denoted as: Yt or 0Yt for emphasis that it starts at t=0 // or 1Y2, 2Y3, etc. to denote starting and ending time
- The **term structure of interest rates** is basically the **pure yield curve** (for zero coupon bonds) made up of the various t spot rates - plot of YTM as a function of time to maturity
  - It is the spot rates that the market sets in equilibrium
  - We can price bonds using the yields or spot rates that are appropriate to its maturity
- We normally observe market prices of bonds, which implies the term structure. Through a process of **bootstrapping**, we can "back out" the spot rates of bonds to find the implied price of bonds.
  - Solve for Y1 with P @ t=1
  - Sub Y1 and solve for Y2 with P @ t=2
  - Etc…
  - CAN SOLVE FOR TERM STRUCTURE BY MATCHING CASH FLOWS
- **Arbitrage opportunities rely on creating synthetic instruments which allow us to exploit mispricings by selling the real instrument and buying the cheaper synthetic instrument, allowing for an arbitrage trade**

- It is possible to use the process of bootstrapping to find the implied price of a bond which can be compared with its market price to identify arbitrage opportunities. By then constructing a synthetic bond, we are able to exploit the mispricings:
  - Get the arbitrage free price
  - Set up arbitrage trade
  - Construct synthetic bond *** this is essentially constructing scaled versions of bonds with the same or shorter maturity, and combining them to replicate the cash flows needed
  - Exploit mispricing
- A reinvestment adds additional risk to the return on investment as the interest rates may fluctuate, leading to a lower price and a HPR (Holding Period Return) of:

$$HPR = \sqrt{\frac{FV + c + c\left(1 + {}_1y_2\right)}{P_0}} - 1$$
;

- **Forward rates are interest rates for investments we agree on today but take place in the future**; in the absence of arbitrage opportunities, the term structure determines all forward rates
  - **1f2** denotes the forward rate valid at t=1 for reinvestment between t=1 and t=2
  - Set up a replicating strategy
- **When our investment horizon (preferred habitat) does not match cash flows, risk arises as we do not know the forward rates and price and subsequently the returns**
  - **Liquidity Risk** - when investment horizon is less than the cash flows
    - The **liquidity premium** is the premium offered to investors to hold bonds whose maturity does not match their horizon
    - The **liquidity preference theory/preferred habitat theory** suggests that the shape of the term structure is determined by the relative horizons of issues and investors;
      - e.g. if issuers tend to have longer horizons than investors, the yield curve will be upward sloping as issuers (selling bonds or borrowing money) will need to offer higher yields to compensate investors for the additional liquidity risk
      - if investors have longer horizons than issuers, the yield curve will be downward sloping as investors will pay higher prices and lower yields so that issuers are compensated for additional reinvestment risk
  - **Reinvestment Risk** - when investment horizon is higher than cash flows
- **Expectations Hypothesis** - theory that market expectations of future interest rates shape the term structure // i.e. all forward rates equals the expectation of future spot rates
  - $${}_sf_t = E\left({}_sy_t\right)$$
  - Example explaining the logic behind this theory:
    - Assume all future interest rates are known & that the y1 < 1y2
    - Invest $1 for 2 years @ y2 & $1 for 1 year @ y1 and another year @ 1y2
    - Cash flows are equivalent given arbitrage free pricing:

$$(1+y_1)(1+{_1y_2}) = (1+y_2)^2$$

- Now, $[(1+y_1)(1+{_1y_2})]^{1/2} = (1+y_2)$; Yields can be written as the geometric average of 1 year spot rates
- In reality, the future spot rate (1y2) isn't know, but expectations [E(1y2)] would generate a similar result

- **Both theories are likely to be at work in the market such that:** $_sf_t = E(_sy_t) + L$
  - where L is the liquidity premium to hold bonds of that maturity
  - Note: if investors were risk neutral, or if there was no uncertainty about future interest rates, there would be no liquidity/reinvestment risk and L = 0; making the EH true

3 - Duration

- **Duration (D) - average time you have to wait for a present value dollar (expressed in YEARS)**
  - Can be thought of as an adjusted time to maturity measure, which takes into the account the "earlier maturity" of coupons
  - Calculated as the **weighted average of "maturity" with "weights" being the ratio of the PV of cash flows at each time to the price** (PV cash flow / Price)
    - **Weights**:

$$w_t = \frac{PV(CF_t)}{\sum_t PV(CF_t)} = \frac{CF_t}{(1+y)^t} \Big/ P$$

    - **"Macaulay" duration:**

$$D = \sum_{t=1}^{T} w_t t = \sum_{t=1}^{T} \frac{PV(CF_t)}{\sum_s PV(CF_s)} t = \sum_{t=1}^{T} \left[\frac{CF_t / (1+y)^t}{P} t \right]$$

  - **Duration of a perpetuity is:**

$$\text{D} = \frac{1+y}{y}$$

- **What affects the duration?**
  - **Yield** - higher yield means that distant cash flows are discounted more, leading to lower weights & duration
  - **Coupon rate** - higher coupon means larger close cash flows and a lower

duration; bondholder receives repayments faster

- **Duration = maturity ; for zero coupon bonds**

- **Time to maturity** - the longer the time to maturity, the longer the average maturity
- **Modified duration (D* = D/(1+y))** - helps to determine how much the price of a bond changes when the interest rate changes
  - Derive
    - $$\frac{\partial P}{P} = -\frac{D}{(1+y)}\partial y = -D^{*}\partial y$$
- Relationship **only applies to small changes in the yield but we can approximate the price changes with non trivial changes in the yield**
  - Using this method, we are **always underestimating the price**
    - The severity of the error **depends on where we are on the price-yield curve because of the convex curve**
  - **Investors value bonds with larger convexity as they allow for favourable asymmetry in price effects of yield changes**; i.e. a small increase in the yield leads to a smaller fall in price compared to a small decrease in the yield which will lead to a larger increase in the price; where price represents the PV to investors
- **Portfolio Duration** - duration of a portfolio is the weighted average of the duration of its components
  - Similar to how the duration of a coupon bond is the weighted average of the duration of a portfolio of zero coupon bonds
- **Immunisation** - balancing the PV of a portfolio to the PV of a liability (asset-liability matching) so that price changes resulting from yield movements are cancelled out
  - Higher yields are beneficial if we have taken on reinvestment risk - we can reinvest our coupons at better interests
  - Lower yields are beneficial when taking on liquidity risk - we can sell for higher prices
  - Process:
    - **Set duration of portfolio equal to duration of liabilities in order to balance out reinvestment and liquidity risk**
- **Rebalancing** - the portfolio chosen to "immunise" liabilities are only valid at a certain point in time given certain parameters; to keep portfolio immune, must keep recalculating - which carries large transactions costs so firms allow for a certain amount of interest rate fluctuations

4. Markowitz Portfolio Theory

- A **utility function** assigns a value to each outcome so that preferred outcomes have higher values
- In our models, utility will depend on wealth as it is too difficult to account for every outcome
  - **Allows us to model insatiation (increasing function (?)) and risk aversion (downward concavity (?))**
- Risk aversion - means that we prefer certain outcomes to stochastic ones
  - Provide a "risk premium" to incentivise risk averse individuals to take on risk