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Linear Algebra Lecture 1:

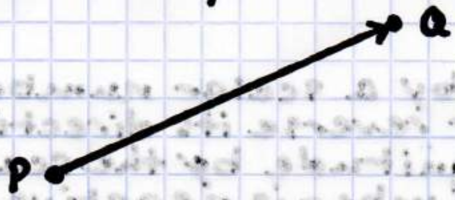
Scalar and Vector Quantities
Vectors given by Points
The Parallelogram
Vector Addition and Subtraction

Scalar and Vector Quantities:
A scalar quantity is a measurement of only magnitude. There is no direction involved. A vector quantity, however, has magnitude and direction. Some examples are in the table below.

Scalars	Vectors
Mass	Velocity
Distance	Force
Temperature	Displacement

Geometric Vectors:
Vector is short for geometric vector, which is a directed line segment in space that is characterised by two properties: its magnitude and its direction. If two vectors have the same magnitude and direction, we consider them equal, regardless of their position in space.

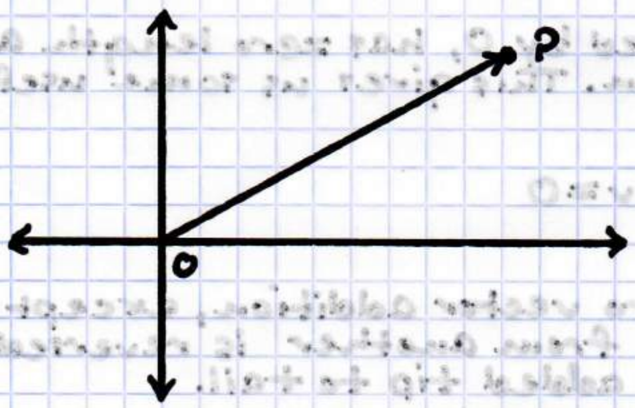
Vectors given by Points:
If P and Q are points in space, then $\overrightarrow{PQ}$ denotes the vector pointing from P to Q.



$|\overrightarrow{PQ}|$ = magnitude

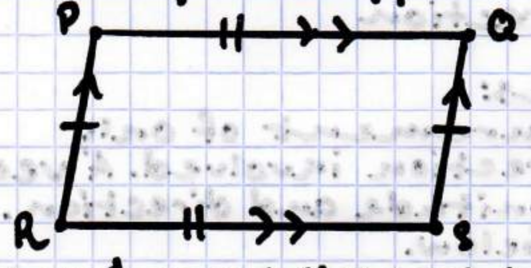
$\overrightarrow{PQ} = -\overrightarrow{QP}$

Position Vectors:
The position vector of a point P is simply the vector from the origin O to P. It is denoted as $\overrightarrow{OP}$.



The Parallelogram:

This is a quadrilateral whose opposite sides are both parallel and equal to each other. However, to prove a quadrilateral is a parallelogram, we only have to show that one pair of opposite sides is equal and parallel.

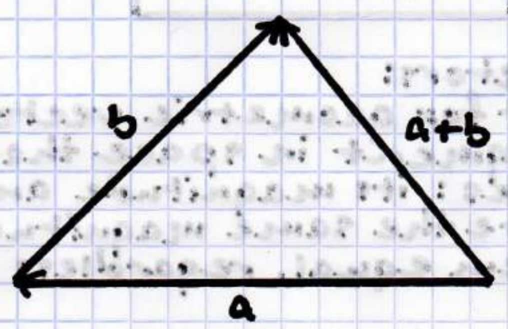


PQRS is a parallelogram

Note: A quadrilateral is a parallelogram if and only if the diagonals bisect each other.

Vector Addition:

To add vectors together, we simply position them from tip to tail as shown below. The sum of vectors is equal to the vector from the tail of the first vector to the tip of the last vector.



Note: The vector sum a+b can be seen as the diagonal of the parallelogram formed using a and b as sides.

Scalar Multiplication:

We can multiply a vector by a scalar number to change its magnitude or reverse its direction. We simply multiply the magnitude by the scalar number. If the scalar number is negative, we simply reverse the direction of the vector.

Zero Vector:

The zero vector, denoted by 0, has zero length and points in every direction. This gives us some useful definitions:

$\overrightarrow{PQ} = -\overrightarrow{QP}$, $v - v = 0$, $0v = 0$

Vector Subtraction:

This is done similarly to vector addition, except that any vector subtracted from another is reversed in direction before being added tip to tail.

Linear Algebra Lecture 2:

Unit Vectors and Hat Notation
Parallel Vectors
Length of a Vector
Linear Independence

Magnitude and Unit Vectors:
A geometric vector v has a length, or magnitude, which is denoted by $|v|$. Note that scalar numbers also have an absolute value as their magnitude $|\lambda|$. We can relate these two concepts by multiplying them.

$$|\lambda v| = |\lambda||v|$$

We call a vector a 'unit vector' if the magnitude is equal to 1.

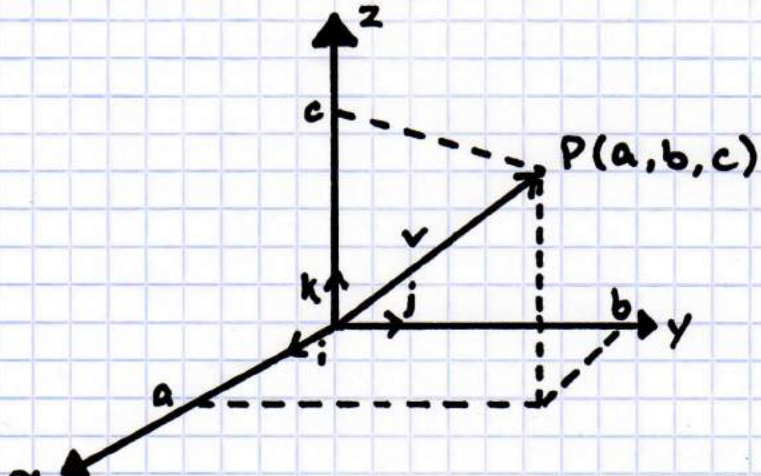
Hat Notation:
The hat of a vector, denoted by $\hat{v}$, is a unit vector pointing in the direction of v. We can define $\hat{v}$ using the following equation.

$$\hat{v} = \frac{v}{|v|}$$

This also means that $v = \hat{v}|v|$.

Parallel Vectors:
Two vectors are said to be parallel if they point in the same or opposite directions. Non-zero vectors are parallel if and only if they are scalar multiples of each other. We can express this with a formula. If v and w are parallel then $v = \lambda w$ for some λ. Also $\hat{v} = \hat{w}$ if $\lambda > 0$. Note that the zero vector is parallel to every vector since it technically points in every direction.

Position Vectors and Components:
We use the letters i, j, k to express the unit position vectors of the x, y, and z axes in 2D or 3D space.



In the plane, if v is the position vector of the point P(a, b) or P(a, b, c), then:
$v = ai + bj$ or
$v = ai + bj + ck$
This is called Cartesian form, and we call the coefficients a, b, and c the components or coordinates of v.

Operations with vectors become much easier when using components. To add vectors together, for example, we simply add the components. The concept is the same when we subtract, negate, or perform scalar multiplication.

If $P(a,b,c)$ and $Q(d,e,f)$, then the position vector $\overrightarrow{PQ}$ is given by:

$$\overrightarrow{PQ} = (d-a)i + (e-b)j + (f-c)k$$

Length of a Vector:
If $v = ai+bj$ and $w = di+ej+fk$, then the length of each is given by:

$$|v| = \sqrt{a^2+b^2}$$

$$|w| = \sqrt{d^2+e^2+f^2}$$

Linear Independence for Two Vectors:
Two vectors x and y are linearly independent if they are not parallel. This also means that if $ax+by=0$, then $a=b=0$, which means that the linear combinations of $v=ax+by$ can produce position vectors whose tips can reach any point in the plane.

Linear Algebra Lecture 3:

Vector Multiplication
Dot Products
Vector Projection

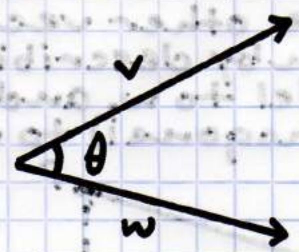
Vector Multiplication:
There are several operations that we can apply to vectors which are considered to multiply them to each other, two of which include:

The scalar or dot product i.e. the scalar quantity $v.w$

The vector or cross product i.e. the vector quantity $v \times w$

Note: we should never just write vw. We need to specify the operation.

Dot Products:
If v and w are geometric vectors in the plane, we label the angle between them when their tails meet θ.



The geometric definition of a dot product is as follows:

$v.w = |v||w|\cos\theta$ which means that $\cos\theta = \frac{v.w}{|v||w|}$.

It is also important to remember that $-1 \leq \cos\theta \leq 1$, therefore the Cauchy-Schwarz Inequality is in effect:

$|v.w| \leq |v||w|$

This also means that if $v.w = |v||w|$ then $\theta = 0°$ and vectors v and w point in the same direction. Similarly, if $v.w = -|v||w|$ then $\theta = 180°$ and vectors v and w point in opposite directions. Finally, if $v.w = 0$ then vectors v and w are mutually perpendicular.

Put simply, if a dot product is positive, then the angle between the two vectors is acute. If negative, the angle is obtuse. And if it is 0, the vectors are perpendiculer to each other.

The algebraic definition of a dot product is as follows:

If $v = ai + bj + ck$ and $w = di + ej + fk$ then
$$v.w = ad + eb + cf$$