JB

MATH3061 Geometry and Topology

Geometry

Orthogonal p.67	A matrix ‘α’ is considered an isometry if its derivative matrix ‘α*’ is orthogonal i.e. matrix has columns of length 1 and are mutually perpendicular.
Determinant	Determinant of a 2x2 matrix i.e. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is ‘ad – bc’ Determinant of a 3x3 matrix i.e. $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ is ‘+ a (ei-fh) – b (di-fg) + c (dh-ge) Note the symbols alternate between + and –
Transpose	When rows of a matrix become columns i.e. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ becomes $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$
Affine transformation	Affine transformations are matrices where the determinant of the derivative matrix ‘α*’ ≠ 0 i.e. If the derivative matrix ‘α*’= $\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix}$, the determinant of ‘α*’ = ad-bc as mentioned above, since determinant of ‘α*’= (2x2) – (4x1) = 0, therefore this is not an affine transformation!
Isometry	Isometries can be categorised into two categories: 1) Even (Determinant of the derivative matrix ‘α*’ = + 1) - Translations (Derivative matrix ‘α*’ maps the identity matrix i.e. $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$) - Rotations (Has one fixed point i.e. x=1, y=5) 2) Odd (determinant of the derivative matrix ‘α*’ = - 1) - Reflections (Has a line of fixed points i.e. y=x) - Glide Reflections (Has no fixed points i.e. 0≠5)

Topology

Euler characteristic of a graph G	The Euler characteristic of a graph G is: X(G)= # Vertices of graph G - # Edges of graph G (Tut 7, definition o)
Euler characteristic of a connected graph	The Euler characteristic of a connected graph is: X(Connected graph) = 1 – “independent circuits”
Euler characteristic of a tree	The Euler characteristic of a tree is: X(Tree) = 1
Euler characteristic of a surface S	The Euler characteristic of a surface S with a polygonal decomposition is: X(S)= #Vertices - #Edges + #Faces (Tut 8, definition i) If two surfaces S and T are connected, then the Euler characteristic of this surface is: X(S#T)= X(S) + X(T) -2 Note that: ‘#’ means connected to (Tut 9, definition g) Furthermore, the Euler characteristic of the surface ($\#^k T \# {}^m P^2$): X($\#^k T \# {}^m P^2$) = k X(T) + m X(P^2) – 2 (k+m-1)