ECON20002 – Intermediate Microeconomics

Study Notes

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1 Intermediate method: optimisation, equilibrium, comparative statics and efficiency

Intermediate microeconomics is a method before it is a catalogue of market structures. Households, firms, and governments are treated as decision makers who choose the best available action given constraints, and markets are treated as systems that must make those choices consistent with one another. The method has four moving parts. *Optimisation* says what an agent does when objectives and constraints are specified. *Equilibrium* says when the collection of those choices can occur together. *Comparative statics* says how the equilibrium objects change when a parameter changes. *Efficiency* asks whether that equilibrium wastes an opportunity for mutually beneficial reallocation. Almost every later topic is a special case of those four.

The usefulness of the method is that it forces the modeller to name the objective, the constraint, and the information the agent is assumed to have. A firm that maximises profit with a known technology and known prices is a different object from a firm that maximises a manager's payoff under hidden effort. A household that maximises utility with a known budget is a different object from a household that cannot distinguish good used cars from lemons. The limitation of the method is the same fact: if the objective, the constraint, or the information is misspecified, the model's comparative statics are the answer to the wrong question. The notes that follow keep returning to that limitation rather than treating it as an afterthought.

Definition
An *economic agent* is a decision maker with an objective (utility, profit, a social welfare function) and a feasible set. A *choice* is an element of the feasible set that is ranked at least as highly as every other feasible element. An *equilibrium* is a collection of choices and prices (or other mediating variables) such that each agent's choice is optimal given the others, and all stated resource or market-clearing conditions hold.

1.1 Constrained optimisation

Unconstrained optimisation is the calculus of an interior peak: if f is differentiable and x^* maximises f on an open set, then $f'(x^*) = 0$, and a second-derivative (or Hessian) test distinguishes maxima from minima and saddles. Economic problems are almost never unconstrained. Income, time, technology, legal rules, and information all restrict the feasible set. The workhorse is therefore *constrained* optimisation.

The standard consumer problem is

$$\max_{x,y} u(x,y) \quad \text{subject to} \quad p_x x + p_y y = m, \quad x \geq 0,\, y \geq 0.$$

When the solution is interior, the Lagrangian

$$\mathcal{L}(x,y,\lambda) = u(x,y) + \lambda\big(m - p_x x - p_y y\big)$$

delivers the first-order conditions

$$u_x = \lambda p_x, \quad u_y = \lambda p_y, \quad p_x x + p_y y = m.$$

Dividing the first two equations yields the tangency condition that the marginal rate of substitution equals the price ratio,

$$\text{MRS} = \frac{u_x}{u_y} = \frac{p_x}{p_y}.$$

The multiplier λ is the marginal utility of income: the rate at which maximised utility rises if the budget is relaxed by one dollar. That reading is an envelope result, not an extra assumption.

Formula / method

Lagrangian recipe (equality constraint). To maximise $f(x)$ subject to $g(x) = 0$, form $\mathcal{L} = f(x) + \lambda g(x)$ (sign conventions differ; what matters is consistency). Set every partial derivative of $\mathcal{L}$ to zero, including $\partial\mathcal{L}/\partial\lambda$, which recovers the constraint. Interpret λ as the shadow price of the constraint: the derivative of the maximised value with respect to a relaxation of g. Check second-order conditions or, in applications, the economic shape of preferences or technology (convexity, diminishing MRS, diminishing MRTS) that makes the critical point a maximum.

Non-negativity and other inequality constraints are handled by Kuhn–Tucker conditions. If a good is not consumed, its marginal utility per dollar cannot exceed that of the goods that are consumed; complementary slackness says that either the quantity is zero or the first-order inequality holds as an equality. A corner choice is not a pathology. It is the right prediction when relative prices, or relative productivities, make one activity unattractive.

Key concept

First-order conditions are *necessary* for an interior optimum under standard smoothness. They are *sufficient* when the objective is concave and the constraint set is convex (quasi-concave utility and a linear budget; concave production and a convex input requirement set). When convexity fails – non-convex preferences, indivisibilities, increasing returns – a critical point can be a minimum or a saddle, and existence of a competitive equilibrium is no longer guaranteed. Convexity is therefore not a mathematical nicety. It is an assumption with economic content, and it is one of the theory's binding limitations.

A second envelope fact is used constantly. Let $v(a)$ be the maximised value of an objective when a parameter a (a price, a tax, a productivity shifter) changes. At an interior optimum the derivative $v'(a)$ equals the partial derivative of the Lagrangian with respect to a, holding the choice variables at their optimal values. The agent is already optimising, so the indirect effect through re-optimised choices is second-order. That is why the derivative of maximised profit with respect to an output price is output (Hotelling's lemma), and why the derivative of the expenditure function with respect to a price is Hicksian demand (Shephard's lemma). Duality in later topics is envelope mathematics with economic names.

1.2 Equilibrium and comparative statics

Optimisation describes one agent. Equilibrium describes a system. In a single competitive market, equilibrium is a price at which quantity demanded equals quantity supplied. In a general equilibrium, it is a whole list of prices such that every market clears and every agent is optimising. In a Cournot oligopoly, it is a pair of outputs such that each firm's output maximises its profit given the other firm's output (a Nash equilibrium). The common idea is consistency of plans: no agent has a feasible, profitable, or utility-raising deviation, and no resource is required that is not supplied.

Definition

Comparative statics is the study of how equilibrium objects (prices, quantities, utilities, profits) change when a parameter changes: an income, a wage, a tax, a cost, a probability, an information structure. A *qualitative* comparative static is a sign (demand falls when own-price rises if the good is ordinary). A *quantitative* comparative static is a magnitude (an elasticity, a pass-through rate, a deadweight-loss triangle).

The cleanest comparative statics come from the structure of the optimisation rather than from brute-force differentiation of every first-order condition. If preferences are homothetic, income expansion paths are rays and Engel curves are linear. If a tax is imposed on a competitive market, the incidence split is governed by the ratio of elasticities, not by which side writes the cheque. If a cost shock hits both Cournot firms equally, both reaction functions shift and the Nash point moves along a predictable path. Method first, algebra second.

Three distinctions keep comparative statics honest. A movement *along* a curve is a response to a change in the variable on the axis. A *shift* of a curve is a response to a change in something else. Holding other things equal is a modelling choice, not a claim that other things are in fact equal. And a comparative-static prediction is only as good as the equilibrium concept: the effect of a cost increase on price is not the same under perfect competition, monopoly, Cournot, and Bertrand, which is why market structure is not an optional extra.