BFC2751

DERIVATIVES

Complete Semester Lecture Notes

Weeks 1–11 · Full lecture coverage · Worked examples, payoff diagrams and formulas

What is inside

- Every lecture from forwards and futures through to the option Greeks
- Fully worked numerical examples with the arithmetic shown step by step
- Payoff diagrams, binomial trees and Black-Scholes calculations
- Real-world cases: Lehman Brothers, Barings Bank, Metallgesellschaft
- Bonus section on exotic and path-dependent derivatives

Contents

Each week begins on a new page.

Unit Overview

BFC2751 Derivatives runs over twelve teaching weeks. Week 6 is the mid-semester test, so there is no Week 6 lecture. The table below maps each week to its lecture topic and to the section of these notes that covers it.

Week	Lecture topic	What it covers
1	Introduction to forwards and futures	Why derivatives exist; forwards vs futures; margining; payoff diagrams
2	Hedging with forwards and futures	Long and short hedges; basis risk; hedge ratios; cross-hedging
3	Valuation of forwards and futures contracts	Cost of carry; income and storage costs; arbitrage bounds; convenience yield
4	Option contracts	Calls and puts; moneyness; intrinsic and time value; option payoffs
5	Hedging with options	Protective puts; covered calls; comparing option and futures hedges
6	Mid-semester test — no lecture	—
7	Option trading strategies	Spreads, straddles, strangles, butterflies; the Barings Bank case
8	Pricing options: Black-Scholes, bounds and put-call parity	Upper and lower bounds; put-call parity; the Black-Scholes-Merton formula
9	Binomial option pricing: replication and delta hedging	Replicating portfolios; delta; one- and multi-step trees
10	Binomial option pricing II: risk-neutral valuation	Risk-neutral probabilities; American options and early exercise
11	Hedging parameters	Delta, gamma, vega, theta and rho; delta and gamma hedging in practice
—	Bonus: exotic and path-dependent derivatives	Lookback, barrier and Asian options; valuation by binomial tree

Week 1 — Introduction to Forwards and Futures

Why do derivatives exist?

- Due to fluctuating gold prices, a mining company cannot be sure what price they will get
- when their gold is extracted and processed ready for sale
- Due to fluctuating exchange rates, an Australian importer who is invoiced in USD will fear a decline in the value of AUD
- Due to volatility in share markets, fund managers fear another market crash like 1987 or 2008 or Feb-2020 which will reduce the value of their portfolios
- Derivative securities enable us to manage these sorts of risks and minimise/eliminate uncertainty

What is a derivative?

- A derivative security is an instrument whose value depends on the value of another variable:
    - The underlying variable might be a traded asset (e.g., stocks, market indices, exchange rates, interest rates, etc.)
    - It might be a commodity (gold, oil, gas wheat, cocoa, pork bellies, orange juices juice)
    - Or, it might be something more exotic (e.g., electricity, rainfall, temperature, carbon emissions)
- Common types of derivatives securities include, forward contracts, future contracts, options and swaps

Are derivatives good or evil?

- Derivative securities/markets are often viewed in a negative light:
- Derivatives markets have not always been tightly regulated.
- Derivatives have occasionally been at the center of high-profile scandals resulting in huge losses.
- The features of derivative securities are often complicated and many people do not properly understand them.

Lehman Brothers bankruptcy:

- Lehman Brothers filed for bankruptcy on 15 Sept-2008. This was the biggest bankruptcy in US history.
- Lehman was an active participant in the OTC derivatives market:
- It took high risks and got into financial difficulty when it was unable to roll over its short-term funding.
- It had hundreds of thousands of transactions outstanding with about 8,000 counterparties.
- Unwinding these transactions has been challenging for both Lehman liquidators and their counterparties.

Are derivatives good or evil?

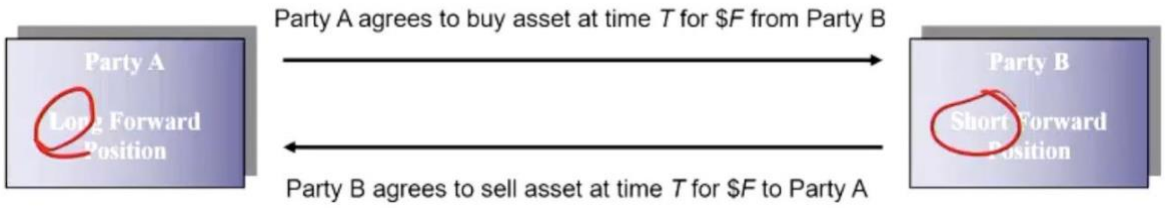
- Nonetheless, derivative markets are here to stay:
- Since options were first traded in Chicago in 1973, the use and trading of derivatives has exploded.
- The value of derivative trading dwarfs the value of trading in the underlying securities.

Forward contracts:

- A forward contract is an agreement to either buy or sell an asset at a specified price on a specified future date.
    - **Aside**: contrast this to a spot contract, which is an agreement to buy or sell an asset today at the current market (or "spot") price S_0.
- Let *F* denote the delivery price for the forward contract and *T* denote the time remaining until delivery.

Always two parties to a forward contract:

- Long forward position: the party who takes a long forward position is contracted to buy the underlying asset
- Short forward position: the party who takes a short forward position is contracted to sell the underlying asset

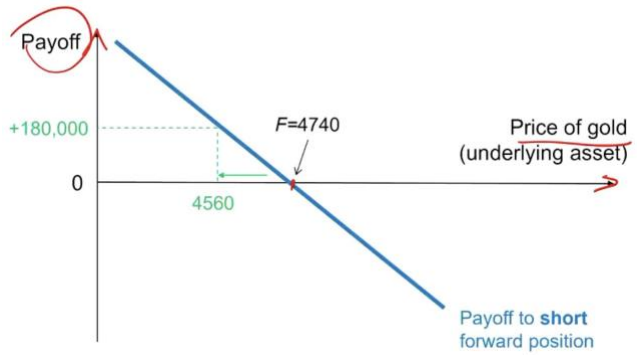


1. Lets say i predict gold to fall to about 4560 by Oct 2026, I contact bank and learn forward price is 4740 USD
    a. Should I long or short forward contracts

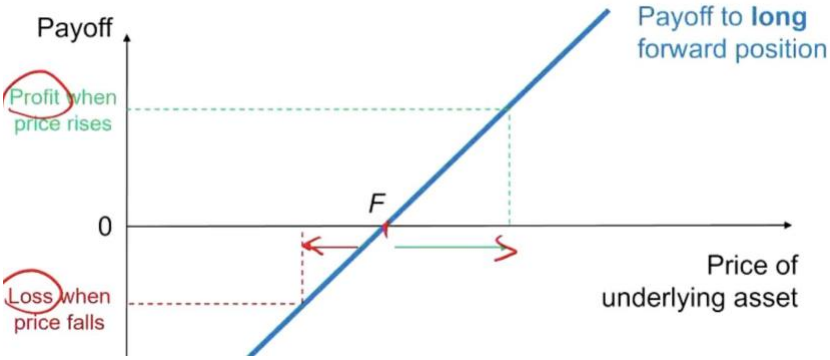
    Since I think the price will be lower than what the bank thinks, I have the choice, do i want to buy gold at that later time at price 4740 or would i rather sell at price 4740, when the market is 4560 (my prediction)
    I would enter a short and as such sell gold at value 4740 in Oct 2026
    If price does fall to 4560 then \i would make $180 profit per ounce
    b. What if price climbs to 5200

    Then I will have to sell gold at price 4740 when market price is 5200, therefor I will loose $460 per ounce

Payoff Diagram: Short forward position



Payoff Diagram: Long forward position



Where do forward contracts trade?

- Forward contracts are traded in the over-the-counter (OTC) market:
  - OTC market is an informal market where traders communicate by phone and/or computer.
  - It is a network of traders working at financial institutions, corporations or fund managers.
- Financial institutions often act as "market makers":
  - When a customer wants to enter a long forward contract, the institution is prepared to take the short side. And vice versa.
  - They quote bid prices (price at which they are prepared to go long).

Defining characteristics of forward contracts:

- Because they are arranged informally between two parties, forward contracts can be highly customised (wrt size, timing, delivery, underlying asset).
- Each party bears some credit risk:
  - Delivery under the contract is not guaranteed.
  - Hence, there is some risk that the counterparty defaults.

Futures Contracts:

- A futures contract is an agreement to either buy or sell an asset at a specified price on a specified future date.
- Let F denote the delivery price for the futures contract and T denote the time remaining until delivery.
- Note that this is the same definition as given for a forward contract!

- In terms of their outcomes and the ways in which they are used, there is a little difference between forwards and futures.

Defining characteristics of future contracts:

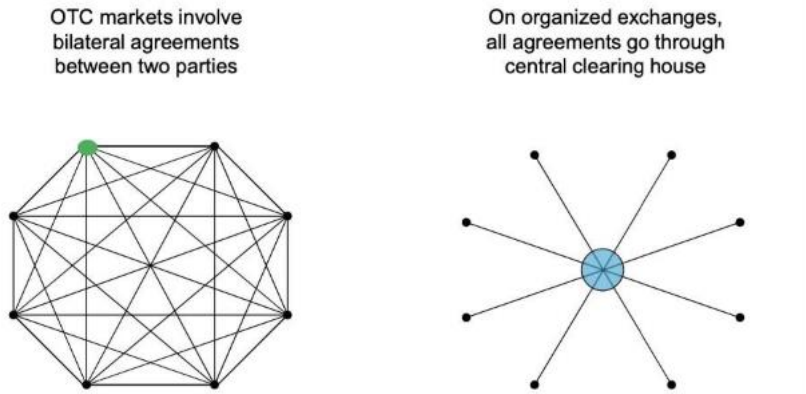
The characteristics of futures contracts that differ most from forward contracts are as follows:

- Future contracts trade on organised exchanges
- There is no risk of default
- Futures contracts are highly standardised
- Futures contracts are "marked to the market" on a daily basis
- Futures contracts are often cash settlements, with no physical delivery of the underlying asset

Organised exchange trading:

Whilst **forwards trade over the counter**, futures contracts trade on **organised exchanges** such as (ASX, Hong Kong exchange, Chicago exchange…)

- Some big advantages of having the contracts trade on an exchange:
- No effort required to find counterparty
- No default risk. With a forward contract, there is always a danger the counterparty will default. With futures contracts, the exchange "clearing house" assumes the role of the counterparty. In essence, the clearing house guarantees the specified terms will be met.



Standardised contract specifications:

- The specific terms of a futures contract are highly standardized with respect to:
- Underlying asset: precisely what asset must be delivered and what its quality must be.
- Contract size: the amount of the underlying asset that has to be delivered under one contract.
- Delivery date: which month the contract expires in.
- Delivery arrangements: if physical delivery is possible, exactly the underlying asset must be delivered.

Marking to the market:

Futures contracts are marked to the market on a daily basis, forward contracts are not.

In brief:

- When you open a futures position (irrespective of whether you go long or short), you must lodge an initial margin (i.e. a deposit) with the exchange
- At the end of each day, the exchange calculates whether you have made or lost money today (i.e. mark you to the market). Gains (losses) are added to (subtracted from) your account
- If your balance gets too low, the exchange makes a margin call (i.e. requires you to deposit more funds).

Settlement procedures for futures:

Once you have entered a futures position (either long or short):

- You might hold that position right through to the expiry/delivery date, or
- You might "close-out" prior to expiry

Let's consider how each of these plays out

Closing-out prior to expiry:

All futures contracts can be "closed-out" prior to expiry

Closing out a futures position simply means entering a new futures position *equal in magnitude but opposite in direction* to your original futures position

- If your opening futures position was long, you close it out by entering a short position in the same futures contract
- If your opening futures position was short, you close it out by entering a long position in the same futures contract

Example: we enter an opening position of one long contract on Jan-2025 Eastern Australian Wheat futures at F = \$422 per tonne. A couple days later:

If the wheat futures price is 600 when we close:

- Close by entering one short Jan-25 wheat contract at 600
- Profit of \$3,560 = (600 - 422) x 20 tonnes

If the futures price is 250 when we close:

- Close by entering one short Jan-25 wheat contract at 250
- Loss is \$3,440 = (250-422) x 20 tonnes

Note: Wheat futures have a standard contract size of 20 metric tonnes

Physical delivery at expiry:

Some (but not many) futures contracts allow physical delivery of the underlying asset on the expiry date.

- The contract specifications clearly articulate where, when and how the delivery must take place
- In such cases, the futures contract is very similar to a forward contract -

In practise, it is rare that futures contracts result in physical delivery:

- The vast majority of futures are closed out prior to expiry
- They have served their purpose (hedging or speculation)
- Physical delivery is more trouble that it is worth

Cash settlement at expiry:

Some futures contracts cannot be physically delivered.

- For example, the asset underlying SPI200 index futures is a portfolio of 200 sticks. It is infeasible to deliver a portfolio of 200 stocks precisely weighted to mimic the S&P/ASX 200 index.

Instead, these futures are cash settled.

- If my futures contract has not been closed out prior to expiry, the exchange effectively enters a reversing transaction on my behalf. For example:
- Assume that my initial trade was to enter 2 long contracts on the SPI200 futures expiring in Sep-2024 (F = 6677 points).
- On the expiry date in September 2024, the ASX effectively takes 2 short Sep-24 SPI200 contracts on my behalf.
- They calculate whether I have made a profit or loss, which is then transferred to/from my account.

Eg 1.

On 22 Jul-24, we enter 2 long Sep-25 SPI200 futures contracts at 6677 (see "settlement price" in table).

a. At expiry, if the Sep-25 SPI200 futures price is 7000:

Close by entering 2 short Sept-25 SPI200 contracts at 7000.

Profit of $16,150 =2 ctts × (7000 – 6677) x $A25.

b. At expiry, if the Sep-25 futures price is 6100:

Close by entering 2 short Sept-25 SPI200 contracts at 6100.

Loss is $28,850 = 2 ctts × (6100 – 6677) x $A25.

Note: SPI200 futures have a standard contract size of $25 per index point

Futures contracts in Australia (ASX):

Equities:

- Shared price index (SPI200) futures -

Interest rates:

- 30-day interbank cash rate
- 90-day bank accepted bill (BAB) futures - 3-year, 10-year and 20-year bond futures

Agricultural:

- Eastern Australia wheat futures, WA wheat futures
- Eastern Australia feed barley futures

Energy futures: some base load electricity contracts

US exchanges offer trading on a much wider variety of underlying assets (CME)

Comparison of forwards and futures:

	Forwards	Futures
Traded where?	Over the counter	On exchanges
Terms of contract are	Tailored to your needs	Highly standardised
Delivery date is	Tailored to your needs	A range of standard delivery months
Settlement occurs	At expiry	Marked to market daily
What happens to the underlying asset?	Often physically delivered	Contract is usually closed out prior to expiry
Any credit risk?	Yes, some	None

Week 2 — Hedging with Forwards and Futures

Speculation v hedging:

Speculation is akin to betting/gambling:

- You have a prediction on the direction of price movements
- You enter a derivatives position that will profit if you guess right
- And, of course, your derivatives position will lose if you guess wrong
- Effectively, a speculator is taking on risk ("putting their money where their mouth is")
- Usually, a speculator does not have any position/exposure in the underlying asset itself

Hedging is about removing risk:

- The hedger is not predicting/guessing which way prices will move
- Rather, they know what price movement will hurt them, and
- They enter a derivatives position to make money if price moves in this unfavourable direction
- Usually, the hedger does have a position/exposure in the underlying asset - Fully hedged = you do not have any loss (but also no profit)!

Basic principles of hedging:

- People hedge when they do not want to be exposed to movements in the price of an asset
- The idea of hedging is to enter a derivatives position that provides a payoff that offsets price movements in the particular asset to which they are exposed
- The aim is to eliminate (or at least reduce) your exposure to the price movements

Strategy:

1. Identify your risk/exposure in the asset
2. Enter a derivatives position that makes money when your unfavourable scenario occurs

Example: Hedging commodity price risk:

My business is in building roads and highways. I have just been awarded a contract to construct a road in China:

- It is a long-term construction project, with the road due to be completed by end 2026.
- Towards the end of the construction process, I will need to purchase about 500 tons of bitumen.
- The current (i.e.,"spot") price of bitumen is RMB 3351 per ton,but I have no idea what the price will be by end 2026.
- With a little investigation, I discover that Shanghai Futures Exchange offers futures contract on bitumen.