MATH1064: Discrete Mathematics for Computation

Complete Exam Preparation Notes with Worked Examples and Practice Examination

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Independent study document compiled from the supplied 2025 unit materials

Contents

1 Sets and Set Operations

Set theory supplies the language used throughout the unit. Functions, relations, graphs, probability spaces, and formal languages are all built from sets, so precise notation at this stage prevents logical errors later. This chapter consolidates the set notation and proof methods used in the supplied lecture, tutorial, practice, and revision materials.

1.1 Sets, elements, and subsets

> **Definition:** A **set** is a collection of distinct objects called **elements**. If an object x belongs to a set A, write $x \in A$; otherwise write $x \notin A$.

Here, x denotes an object and A denotes a set.

> **Definition:** A set A is a **subset** of a set B, written $A \subseteq B$, when every element of A is also an element of B.

Here, A and B denote sets.

The empty set, written $\emptyset$, contains no elements. It is a subset of every set because there is no element of $\emptyset$ that can violate the subset condition. Set equality is stronger than equal cardinality: two sets are equal only when they contain exactly the same elements.

The standard proof of set equality is

$$A = B \iff A \subseteq B \text{ and } B \subseteq A. \tag{1.1}$$

where:

- A and B are the sets being compared;
- $\subseteq$ denotes the subset relation.

Membership and containment must not be confused. If $A = \{1, 2, 3\}$, then $2 \in A$, while $\{2\} \subseteq A$. The statement $\{2\} \in A$ is false because the elements of A are numbers, not singleton sets.

1.2 Set-builder notation and operations

Set-builder notation describes elements by a property:

$$A = \{x \in X : P(x)\}. \tag{1.2}$$

where:

- X is the ambient set or domain;
- x is a candidate element of X;
- $P(x)$ is the property required for membership in A.

Let A and B be subsets of a universal set U. The main operations are summarised below.

Operation	Notation	Membership condition
Union	$A \cup B$	$x \in A$ or $x \in B$
Intersection	$A \cap B$	$x \in A$ and $x \in B$
Difference	$A \setminus B$	$x \in A$ and $x \notin B$
Complement	A^c	$x \in U$ and $x \notin A$
Symmetric difference	$A \triangle B$	x belongs to exactly one of A and B

The complement depends on the chosen universal set U. A statement involving A^c is incomplete if the ambient universe has not been established.

1.2.1 Worked example 1.1: computing set operations

Let

$$U = \{1, 2, 3, 4, 5\}, \qquad A = \{1, 2, 4\}, \qquad B = \{2, 3\}.$$

where U is the universal set and $A, B \subseteq U$. Then

$$A \cup B = \{1, 2, 3, 4\}, \qquad A \cap B = \{2\},$$

$$A \setminus B = \{1, 4\}, \qquad A^c = \{3, 5\}.$$

The union collects elements occurring in at least one set. The intersection retains only common elements. The difference begins with A and removes elements of B, while the complement removes A from the declared universe.

1.3 Set identities and proofs

Law	Union form	Intersection form
Identity	$A \cup \emptyset = A$	$A \cap U = A$
Domination	$A \cup U = U$	$A \cap \emptyset = \emptyset$
Idempotent	$A \cup A = A$	$A \cap A = A$
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$

1.3.1 Worked example 1.2: an element-chasing proof

Prove

$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C). \tag{1.3}$$

where A, B, and C are subsets of the same universe. Choose arbitrary x in that universe. Then

$$\begin{aligned} x \in A \setminus (B \cup C) &\iff x \in A \text{ and } x \notin B \cup C \\ &\iff x \in A \text{ and } x \notin B \text{ and } x \notin C \\ &\iff x \in A \setminus B \text{ and } x \in A \setminus C \\ &\iff x \in (A \setminus B) \cap (A \setminus C). \end{aligned}$$

Because the membership conditions are equivalent for every x, the sets are equal.

1.4 Power sets and Cartesian products

Definition: The **power set** of A, written $\mathcal{P}(A)$, is the set of all subsets of A.

If A has n elements, then

$$|\mathcal{P}(A)| = 2^n. \tag{1.4}$$

where $n = |A|$ is the number of elements of the finite set A. Each element has two independent states: included or excluded.

Definition: The **Cartesian product** of A and B is

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}.$$

where a is an element of A and b is an element of B.

For finite sets,

$$|A \times B| = |A|\,|B|. \tag{1.5}$$

where $|A|$ and $|B|$ are the cardinalities of the factor sets. Ordered pairs retain position, so (a, b) and (b, a) need not be equal. This distinction becomes essential when Cartesian products define functions and relations.

Chapter recap: Prove set equality by double inclusion or a chain of equivalent membership statements. Keep $\in$ separate from $\subseteq$, state the universe before taking complements, and identify whether the elements under discussion are objects, sets, or ordered pairs.

2 Functions and Sequences

Functions formalise assignments between sets and reappear in relations, countability, and finite-state models. Sequences are functions with integer index sets, so the sequence material extends the same language. The central exam habit is to work from the declared domain and codomain rather than from a formula alone.