FINS2624

Portfolio Management

Comprehensive Course Notes

UNSW Business School

Course Structure

Part 1 (Weeks 1-3): Returns, Risk, and Portfolio Theory
Week 1: Returns, expected returns, variance, covariance, correlation, portfolio risk
Week 2: Mean-variance frontier, efficient frontier, two-fund separation, CAL, Sharpe ratio
Part 2 (Weeks 3-5): Asset Pricing Models
Week 3/4: Capital Asset Pricing Model (CAPM), SML, beta, risk decomposition
Week 4/5: Single Index Model (SIM), alpha, active portfolio construction, SIM vs CAPM
Week 5: Fama-French-Carhart 4-factor model, SMB, HML, MOM
Part 3 (Weeks 6-7): Market Efficiency and Behavioural Finance
Week 6/7: EMH (weak, semi-strong, strong), anomalies, overconfidence, under-diversification
Part 4 (Weeks 8-9): Fixed Income Securities
Week 8: Bond pricing, YTM, coupon bonds, no-arbitrage pricing, holding period return
Week 9: Term structure of interest rates, forward rates, duration, immunisation
Part 5 (Week 10): Derivatives and Options
Week 10: Options payoffs, binomial model, risk-neutral pricing, Black-Scholes, put-call parity

WEEKS 1-2: RETURNS, RISK, AND PORTFOLIO THEORY

1.1 Returns

The return on an asset over a holding period is the total gain expressed as a proportion of the initial investment, including both price appreciation and any income (dividends). Returns are the fundamental unit of measurement in portfolio management.

Core Return Formulas

Holding Period Return: r = (P_T - P_0 + D) / P_0
  where P_T = ending price; P_0 = beginning price; D = dividends received

Annualising (from monthly data):
  Annualised mean return = monthly mean x 12
  Annualised std dev = monthly std dev x sqrt(12)
  Annualised variance = monthly variance x 12
  Annualised covariance = monthly covariance x 12

Arithmetic average: r_bar = (1/T) x Sum(r_t) [best estimate of next period's expected return]
Geometric average: r_geo = [(1+r_1)(1+r_2)...(1+r_T)]^(1/T) - 1 [compound historical rate]

1.2 Expected Return (Forward-Looking)

Expected return uses probability-weighted outcomes for forward-looking analysis. Each scenario has a probability, and the expected return is the probability-weighted sum of scenario returns.

Expected Return

E[r] = Sum_s [pi_s x r_s] where Sum(pi_s) = 1

Example: 3 states; pi_1=0.25,r_1=6%; pi_2=0.50,r_2=14%; pi_3=0.25,r_3=18%
E[r] = 0.25*0.06 + 0.50*0.14 + 0.25*0.18 = 0.1300 = 13.00%

Portfolio expected return = weighted average of component expected returns:
E[r_p] = Sum_i [omega_i * E[r_i]] where Sum(omega_i)=1
Portfolio return is always linear in the portfolio weights.

1.3 Variance, Standard Deviation, and Covariance

Risk Measures: Variance, Std Dev, Covariance, Correlation

Variance (probability-weighted): sigma^2 = Sum_s [pi_s * (r_s - E[r])^2]
Variance (sample): s^2 = (1/(T-1)) * Sum_t [(r_t - r_bar)^2] [use T-1 for unbiasedness]
Std deviation (volatility): sigma = sqrt(sigma^2)
Excel: =VAR.S() for variance; =STDEV.S() for std dev

Covariance: Cov(r_i,r_j) = Sum_s[pi_s*(r_i,s-E[r_i])*(r_j,s-E[r_j])]
Sample covariance: (1/(T-1))*Sum_t[(r_i,t - r_bar_i)*(r_j,t - r_bar_j)]
  Cov(r_i,r_i) = Var(r_i) = sigma_i^2 [special case: covariance with itself]

Correlation: rho_{ij} = Cov(r_i,r_j) / (sigma_i * sigma_j) [-1 <= rho <= +1]
Restating: Cov(r_i,r_j) = rho_{ij} * sigma_i * sigma_j
rho=+1: perfect positive; rho=0: uncorrelated; rho=-1: perfect negative

1.4 The Covariance Matrix and Portfolio Variance

The N x N covariance matrix completely describes the risk relationships among N assets. Diagonal elements are variances; off-diagonal elements are pairwise covariances. Portfolio variance is computed using the cell-by-cell multiplication of weights and the covariance matrix.

Covariance Matrix (3-Asset Portfolio)

Label rows and columns with portfolio weights (omega_1, omega_2, omega_3):

	omega_1 col	omega_2 col	omega_3 col
omega_1	sigma_1^2	rho12*s1*s2	rho13*s1*s3
omega_2	rho12*s1*s2	sigma_2^2	rho23*s2*s3
omega_3	rho13*s1*s3	rho23*s2*s3	sigma_3^2

Portfolio variance = multiply each cell by its row-weight AND column-weight, then sum:
Var(r_p) = omega_1^2*sigma_1^2 + omega_2^2*sigma_2^2 + omega_3^2*sigma_3^2
+ 2*omega_1*omega_2*rho12*sigma_1*sigma_2
+ 2*omega_1*omega_3*rho13*sigma_1*sigma_3
+ 2*omega_2*omega_3*rho23*sigma_2*sigma_3

This generalises to any N assets. The key insight: off-diagonal terms reduce portfolio variance when correlations are less than 1 -- this is the diversification benefit.

1.5 Covariance Between Two Portfolios

The covariance between two different portfolios P and Q with weights omega and psi is computed the same way: label rows with portfolio P weights and columns with portfolio Q weights, then multiply each cell by its row weight and column weight and sum.

Covariance Between Two Portfolios

Cov(r_P, r_Q) = Sum_i Sum_j [omega_i * psi_j * Cov(r_i, r_j)]

Example: Portfolio P is equally-weighted in all 3 assets (omega=1/3 each);
Portfolio Q is equally-weighted in assets 1 and 3 only (psi_1=psi_3=1/2, psi_2=0).

Cov(P,Q) = (1/3)(1/2)*sigma_1^2 + (1/3)(0)*Cov(1,2) + (1/3)(1/2)*Cov(1,3)
+ (1/3)(1/2)*Cov(2,1) + 0 + (1/3)(1/2)*Cov(2,3)
+ (1/3)(1/2)*Cov(3,1) + 0 + (1/3)(1/2)*sigma_3^2

Three-Asset Portfolio Worked Example

E[r_1]=6%,E[r_2]=14%,E[r_3]=18%; sigma_1=22%,sigma_2=15%,sigma_3=20%

rho_12=0.50, rho_13=0.35, rho_23=0.25; equal weights omega=1/3

E[r_p] = (1/3)*6% + (1/3)*14% + (1/3)*18% = 12.67%

Var(r_p) = (1/3)^2*(0.22)^2 + (1/3)^2*(0.15)^2 + (1/3)^2*(0.20)^2
        + 2*(1/9)*0.50*0.22*0.15 + 2*(1/9)*0.35*0.22*0.20 + 2*(1/9)*0.25*0.15*0.20
        = (1/9)[0.0484 + 0.0225 + 0.0400 + 0.0330 + 0.0308 + 0.0150]
        = (1/9)[0.1897] = 0.02108
sigma_p = sqrt(0.02108) = 14.52% (note: problem solutions use 14.41% depending on rounding)

Diversification benefit vs all in Asset 1: 22.00% - 14.52% = 7.48%

WEEKS 2-3: MEAN-VARIANCE FRONTIER AND CAPITAL ALLOCATION

2.1 Mean-Variance Utility and Risk Aversion

Investors are assumed to maximise mean-variance utility, caring only about expected return and variance. The risk aversion coefficient A determines how much an investor penalises variance.

Mean-Variance Utility

U(r) = E[r] - (1/2) * A * sigma^2
  A > 0: risk averse (standard investor)
  A = 0: risk neutral (maximises return only)
  A < 0: risk loving

Higher A = steeper indifference curves = more return needed per unit of risk

Dominance: Portfolio B dominates A if B has BOTH higher return AND lower risk.
All investors (any A>0) prefer B over A in that case.

Finding A for indifference between portfolios P and Q:
Set U(P) = U(Q): E[r_P] - (1/2)*A*sigma_P^2 = E[r_Q] - (1/2)*A*sigma_Q^2
Solve: A = 2*(E[r_Q]-E[r_P]) / (sigma_Q^2 - sigma_P^2)

2.2 The Minimum Variance Frontier and Efficient Frontier

The minimum variance frontier (MVF) is the boundary of the set of feasible risk-return combinations. For each expected return level, the MVF portfolio minimises variance. The efficient frontier is the upper portion of the MVF: portfolios that cannot be improved upon (no portfolio with the same risk but higher return exists).

MVF and Efficient Frontier Key Concepts

Global Minimum Variance Portfolio (GMVP): the single portfolio with the lowest variance across all possible portfolios. The starting point of the efficient frontier.
Efficient Frontier: the set of MVF portfolios at or above the GMVP. Every rational risk-averse investor selects from the efficient frontier.
Inefficient region: MVF portfolios below the GMVP. For each, there is an efficient portfolio with the same risk but higher return.
Long-short portfolios: allowing short positions extends the frontier further, generally achieving better Sharpe ratios but requiring the ability to short-sell.
iLab context: Markowitz optimisation uses Excel Solver to minimise sigma^2 subject to target return and weight constraints, tracing out the MVF at each target return level.

2.3 Capital Allocation Line and Sharpe Ratio

Combining a risky portfolio P with the risk-free asset produces the Capital Allocation Line (CAL). The slope of the CAL is the Sharpe ratio, which measures reward per unit of total risk. All mean-variance investors prefer the risky asset with the highest Sharpe ratio, as it provides the best CAL.