WEEK 1: DEBT VALUATION

1.1 Bonds and Fixed Income Securities

A bond is a debt security in which the issuer (borrower) promises to make periodic coupon payments to the bondholder and to repay the face (par) value at maturity. Bonds are priced by discounting all future cash flows at the appropriate yield to maturity. The key inputs for bond pricing are the face value, the coupon rate, the yield to maturity (YTM), the maturity date, and the compounding frequency.

Face Value (FV)	The par value of the bond, typically $1,000. This is the amount repaid at maturity.
Coupon Rate (CR)	The annual interest rate stated on the bond, expressed as a percentage of face value.
Yield to Maturity (YTM)	The discount rate that equates the present value of all future cash flows with the current market price. The market's required rate of return.
Credit Spread	The additional yield above the risk-free rate required to compensate for default risk. YTM = Risk-free rate + Credit spread.

1.2 Zero-Coupon Bonds

A zero-coupon bond (ZCB) pays no periodic coupon. It is issued at a discount to face value and redeemed at face value at maturity. The yield on a ZCB for a given maturity is the risk-free spot rate for that term. These yields form the zero-coupon yield curve, which is the basis for pricing all other fixed-income securities.

Zero-Coupon Bond Pricing

$$\text{Price} = FV / (1 + YTM)^n$$

where: FV = face value, YTM = yield to maturity (annual), n = years to maturity

Example: 2-year ZCB with FV = $100, YTM = 5.49%
$$\text{Price} = 100 / (1.0549)^2 = 100 / 1.1129 = \$89.86$$

Rearranging to find YTM from price:
$$YTM = (FV / \text{Price})^{(1/n)} - 1$$

1.3 Coupon Bond Pricing

A coupon bond pays periodic interest plus the face value at maturity. The price is the present value of an annuity (the coupons) plus the present value of the face value. When coupons are paid more than once per year, the rate and periods must be adjusted for the compounding frequency m.

Coupon Bond Formula

Price = CPN x [(1 - (1 + r)^(-n)) / r] + FV / (1 + r)^n

where: r = periodic rate = YTM / m
n = total periods = Years x m
CPN = periodic coupon = (CR / m) x FV
m = compounding frequency (1 = annual, 2 = semi-annual, 4 = quarterly)

Step-by-step process:
Step 1: Calculate r = YTM / m
Step 2: Calculate n = T x m
Step 3: Calculate CPN = (CR / m) x FV
Step 4: Price = CPN x PVA(r, n) + FV / (1+r)^n

Excel: =PV(r, n, CPN, FV) [note: result will be negative; use -PV(...)]

Worked Example: Coupon Bond

Cola Corporation: FV = $7,000, CR = 4%, semi-annual (m=2), T = 5 years, YTM = 8%

Step 1: r = 8% / 2 = 4% per period
Step 2: n = 5 x 2 = 10 periods
Step 3: CPN = (4% / 2) x 7,000 = $140 per period
Step 4: PVA(4%, 10) = [1 - (1.04)^(-10)] / 0.04 = 8.1109
PV(coupons) = 140 x 8.1109 = $1,135.52
PV(face value) = 7,000 / (1.04)^10 = $4,729.03
Bond Price = $1,135.52 + $4,729.03 = $5,864.55

Key insight: Bond trades at discount because YTM (8%) > CR (4%).

1.4 Pricing Using the Zero-Coupon Yield Curve

The zero-coupon yield curve gives the spot rate for each maturity. To price a coupon bond more precisely, each cash flow is discounted at the spot rate appropriate to its maturity date, rather than using a single flat YTM. This is the most theoretically correct pricing method.

Bond Pricing with Spot Rates

Price = C1/(1+y1)^1 + C2/(1+y2)^2 + ... + (CPN+FV)/(1+yn)^n

where yt is the spot rate for maturity t.

Example: FV = $700, CR = 5% annual, T = 4 years
Spot rates: y1=5.02%, y2=5.49%, y3=5.72%, y4=5.98%
CPN = 5% x 700 = $35 per year
PV = 35/(1.0502) + 35/(1.0549)^2 + 35/(1.0572)^3 + 735/(1.0598)^4
= 33.33 + 31.45 + 29.62 + 582.63 = $677.03

1.5 Junk Bonds and Credit Spreads

Investment-grade bonds are issued by firms with strong credit ratings and have low credit spreads. Speculative-grade (junk or high-yield) bonds are issued by firms with lower credit ratings and carry a higher credit spread to compensate investors for default risk. The total YTM = risk-free rate + credit spread. All pricing uses the same formula; only the YTM differs.

Worked Example: Junk Bond

Removalists of Junk Inc.: FV = \$1,000, CR = 14%, semi-annual, T = 13 years
Risk-free rate = 1.8%, Credit spread = 7.1%, so YTM = 8.9%

r = 8.9% / 2 = 4.45% per period; n = 13 x 2 = 26 periods
CPN = (14% / 2) x 1,000 = \$70 per period
PVA(4.45%, 26) = 15.2272 (from annuity table)
PV(coupons) = 70 x 15.2272 = \$1,065.90
PV(face value) = 1,000 / (1.0445)^26 = \$322.39
Bond Price = \$1,065.90 + \$322.39 = \$1,388.29

Note: Bond trades at PREMIUM because YTM (8.9%) < CR (14%).

1.6 Loan Mechanics and Refinancing

A standard loan is structured as an ordinary annuity: the borrower makes equal periodic payments that cover both interest and principal. The periodic payment is calculated by dividing the loan value by the annuity factor. After any given payment, the outstanding balance equals the present value of the remaining payments at the original interest rate. Refinancing involves calculating the outstanding balance at the refinancing date, then computing new payments at the new rate.

Loan Payment and Refinancing Formulas

Monthly payment C = Loan Value / PVA(r, n)
PVA(r, n) = [1 - (1+r)^(-n)] / r

Outstanding balance after k payments = C x PVA(r, n - k)
= PV of remaining payments at the original rate

New payment after refinancing = Outstanding Balance / PVA(r_new, n_remaining)

Excel: =PMT(r, n, -PV) for payment; =PV(r, n_remaining, -PMT) for balance

Worked Example: Loan Refinancing

Jenny: \$40,000 loan at 6% compounded monthly (r = 0.5%), T = 2 years (n = 24)

Original monthly payment = 40,000 / PVA(0.5%, 24) = 40,000 / 22.5629 = \$1,772.82

After 6 payments (18 months remaining):
Outstanding balance = 1,772.82 x PVA(0.5%, 18) = 1,772.82 x 17.1728 = \$30,444.30

Refinance at 5% (r_new = 5%/12 = 0.4167%), 18 months remaining:

PVA(0.4167%, 18) = 17.3453
New payment = 30,444.30 / 17.3453 = $1,755.78/month

Saving per month = $1,772.82 - $1,755.78 = $17.04

1.7 Rate of Return and Reinvestment Risk

When a bondholder sells the bond before maturity, the actual rate of return depends on: the price at which the bond is sold (determined by the YTM at that time), and the rate at which coupons received were reinvested. If the reinvestment rate differs from the original YTM, the actual return will differ from the YTM. This is reinvestment risk.

Rate of Return with Reinvestment

Step 1: Calculate FV of reinvested coupons at reinvestment rate R:
    FV_coupons = CPN x FVA(R, T_hold)
    FVA(R, T) = [(1+R)^T - 1] / R

Step 2: Calculate bond price at the time of sale using YTM2 (new YTM at time of sale):
    P_sell = CPN x PVA(YTM2, remaining n) + FV/(1+YTM2)^(remaining n)

Step 3: Total FV = FV_coupons + P_sell

Step 4: Rate of return = (Total FV / P0)^(1/T_hold) - 1

Worked Example: Rate of Return

Bond: FV=$1,000, CR=12% annual, YTM=12% at purchase (trading at par, so P0=$1,000)
Held 5 years, then sold when YTM = 15%; coupons reinvested at 10%.
CPN = 12% x 1,000 = $120/year

Step 1: FV of reinvested coupons:
FVA(10%, 5) = [(1.10)^5 - 1] / 0.10 = 6.1051
FV_coupons = 120 x 6.1051 = $732.61

Step 2: Sale price (remaining 5 years at YTM2 = 15%):
PVA(15%, 5) = 3.3522; P_sell = 120 x 3.3522 + 1000/(1.15)^5 = 402.26 + 497.18 = $899.44

Step 3: Total FV = 732.61 + 899.44 = $1,632.05
Step 4: Return = (1,632.05 / 1,000)^(1/5) - 1 = 10.29%

Note: Return (10.29%) < original YTM (12%) because the bond was sold at a loss
(YTM rose from 12% to 15%) and coupons were reinvested at only 10%.

WEEK 2: EQUITY VALUATION

2.1 Equity Valuation Fundamentals

The value of a share is the present value of all future cash flows that the shareholder expects to receive. The most direct form of cash flow to equity is dividends. The dividend discount model (DDM) values shares by discounting expected future dividends at the equity cost of capital (re). This framework assumes that value comes from cash distributions to shareholders, which may be dividends or share repurchases.

Total equity return has two components: the dividend yield (D1/P0) and the capital gains yield (P1-P0)/P0. In equilibrium, the required return re equals the expected total return:

One-Period Equity Valuation

$P_0 = (D_1 + P_1) / (1 + r_e)$

$r_e = D_1/P_0 + (P_1 - P_0)/P_0$ = Dividend Yield + Capital Gains Yield

For all periods, P_0 = sum of $[D_t / (1+r_e)^t]$ for t = 1 to infinity

2.2 Gordon Growth Model (Constant Dividend Growth)

When dividends grow at a constant rate g in perpetuity and $g < r_e$, the infinite sum simplifies to the Gordon Growth Model. This is the most widely used equity valuation model for mature, stable companies with predictable dividend growth. D0 is the most recently paid dividend; D1 is the next expected dividend.

Gordon Growth Model

$P_0 = D_1 / (r_e - g)$
$D_1 = D_0 \times (1 + g)$

Required: $g < r_e$ (otherwise the model produces a negative or infinite value)

Rearranging to find re: $r_e = D_1/P_0 + g$
Rearranging to find g: $g = r_e - D_1/P_0$

Interpretation: P0 rises if g increases, D1 increases, or re decreases.

Worked Example: Gordon Growth

Firm just paid D0 = \$4.00; g = 2.0%; re = 10.2%
D1 = 4.00 x 1.02 = \$4.08
P0 = 4.08 / (0.102 - 0.02) = 4.08 / 0.082 = \$49.76

Expected return decomposition:
Dividend yield = 4.08 / 49.76 = 8.2%
Capital gains yield = g = 2.0%
Total = 10.2% = re (consistent with equilibrium)

2.3 Two-Stage Dividend Discount Model

Many companies go through high-growth phases before settling to a long-run stable growth rate. The two-stage model values the high-growth phase as a growing annuity and the stable phase as a growing perpetuity (terminal value). The terminal value is calculated at the end of the high-growth period and then discounted back to the present.

Two-Stage DDM Formula

Stage 1 (High growth): Growing annuity for n years at growth rate g1
P0_annuity = (D1 / (re - g1)) x [1 - ((1+g1)/(1+re))^n]

Stage 2 (Terminal value): Growing perpetuity from year n+1 at rate g2
Pn = D(n+1) / (re - g2)   where D(n+1) = D0 x (1+g1)^n x (1+g2)
P0_perpetuity = Pn / (1 + re)^n

P0 = P0_annuity + P0_perpetuity

Note: D(n+1) = next dividend after the high-growth phase ends

Worked Example: Two-Stage DDM

Amiga Corp: D0 = $0.91, g1 = 10.5% for 5 years, g2 = 5.3%, re = 8.5%

D1 = 0.91 x 1.105 = $1.005

Stage 1 (growing annuity, 5 years):
P0_annuity = (1.005 / (0.085 - 0.105)) x [1 - (1.105/1.085)^5]
= (1.005 / (-0.02)) x [1 - (1.01843)^5]
= -50.25 x [1 - 1.0965] = -50.25 x (-0.0965) = $4.849

Stage 2 (terminal value):
D6 = 0.91 x (1.105)^5 x 1.053 = 0.91 x 1.6474 x 1.053 = $1.579
P5 = 1.579 / (0.085 - 0.053) = 1.579 / 0.032 = $49.34
P0_perpetuity = 49.34 / (1.085)^5 = 49.34 / 1.5037 = $32.81

P0 = 4.849 + 32.81 = $37.66

2.4 Three-Stage Dividend Discount Model

For companies moving through three distinct growth phases (e.g., high growth, transitioning, and stable), a three-stage model is used. The approach is the same as the two-stage model but with an intermediate growth period. Each stage's dividends are valued as a growing annuity and the terminal value is computed at the end of the last transition phase.

2.5 Total Payout Model

When a firm uses both dividends and share repurchases to return cash to shareholders, total payouts replace dividends in the valuation formula. The total payout model values the entire equity (total market