Week 2

Principles of Finance — Week 2: Time Value of Money

Overview

Week 2 developed the core mathematical foundation of finance: the **time value of money (TVM)**. The main idea is simple but extremely important:

> **A dollar today is worth more than a dollar in the future.**

Why? Because money received today can be **invested**, earn a **return**, and grow over time. This week focused on how to move values **forward in time** (future value) and **backward in time** (present value), and how to apply those ideas to **single cash flows**, **multiple cash flows**, **annuities**, **perpetuities**, and **loan-style payment streams**.

These concepts are foundational for almost every topic later in finance — including **bond valuation**, **share valuation**, **project appraisal**, and **portfolio decisions**.

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1. The Time Value of Money

Core principle

Money has value across time because it has an **opportunity cost**. If you receive money now, you can invest it and earn interest or a return.

That means:

- receiving **cash earlier** is usually better than receiving it later,
- paying **cash later** is usually better than paying it now,
- finance often involves comparing cash flows that occur at **different dates**.

Key term definitions

- **Time value of money (TVM)** — the idea that money available today is worth more than the same amount in the future.

- **Opportunity cost** — the return you give up by not using funds in their next-best alternative.
- **Cash flow** — money received or paid at a particular point in time.

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2. Future Value (FV)

What is future value?

Future Value (FV) tells you how much a cash flow today will be worth at a future date if it earns interest or a return over time.

Formula for a single cash flow

$$FV_n = PV(1+r)^n$$

where:

- FV_n = value at time n
- PV = amount invested today
- r = interest rate per period
- n = number of periods.

Interpretation

This formula shows **compounding**:

- in the first period, you earn interest on the original amount,
- in later periods, you earn interest on **both the original amount and previous interest**.

Key term definitions

- **Future value (FV)** — value of money at a future date.
- **Compounding** — earning returns on previous returns.
- **Growth factor** — the term $(1+r)^n$ that scales money forward through time.

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3. Present Value (PV)

What is present value?

Present Value (PV) tells you what a future cash flow is worth **today**.

This is one of the most important concepts in all of finance because most assets are valued by finding the present value of expected future cash flows.

Formula for a single future cash flow

$PV = \frac{FV_n}{(1+r)^n}$

Interpretation

Present value is the reverse of future value. It answers questions like:

- What is **$1,000 in 3 years** worth today?
- How much should I pay today for a future payment?
- How do I compare cash flows at different dates?

Key term definitions

- **Present value (PV)** — current value of a future cash flow.
- **Discounting** — converting future cash flows into today’s dollars.
- **Discount rate** — required rate of return used to discount cash flows.

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4. Discount Rate and Required Return

Why discounting matters

When you discount a future cash flow, you are adjusting it for:

- **time**, and
- the **required return** investors demand.

Key finance interpretation

A higher discount rate means:

- future cash flows are worth **less today**.

A lower discount rate means:

- future cash flows are worth **more today**.

Why this matters

This idea becomes crucial later when pricing:

- **bonds**,
- **shares**,
- **projects**,
- and any stream of future payments.

Key term definitions

- **Required return** — the return investors require for waiting and bearing risk.
- **Discount factor** — the term $1/(1+r)^n$ used to convert future money into present money.

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5. Timeline Technique

A major practical skill in TVM is laying out cash flows on a **timeline**.

Why timelines matter

Timelines help you:

- place each cash flow at the correct date,
- avoid mistakes with timing,
- distinguish between **today (time 0)** and future periods.

Common timing convention

- **Time 0** = today
- **Time 1** = one period from today
- **Time 2** = two periods from today
- etc.

Important habit

In finance, many mistakes come not from formulas, but from putting cash flows at the **wrong time**.

Key term definitions

- **Timeline** — visual layout of cash flows through time.
- **Time 0** — today / present date.

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6. Multiple Cash Flows

In real finance problems, you usually deal with **more than one cash flow**.

Core rule

You cannot directly add or compare cash flows that occur at different times until you move them to the **same date**.

That means you either:

- compound everything to a **future date**, or
- discount everything back to **today**.

Present value of multiple cash flows

$$PV = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \cdots$$

Future value of multiple cash flows

Each cash flow must be compounded separately to the target date.

Key term definitions

- **Cash flow stream** — a sequence of cash flows over time.
- **Valuation date** — the date to which all cash flows are moved for comparison.

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7. Simple Interest vs Compound Interest

Simple interest

Under **simple interest**, interest is earned only on the original principal.

$$FV = PV(1 + rn)$$

Compound interest

Under **compound interest**, interest is earned on:

- the original principal, and

- accumulated interest from prior periods.

$$FV = PV(1+r)^n$$

Key finance conclusion

In almost all real financial applications, **compound interest** is the relevant concept.

Why this matters

Over short periods, the difference may look small.

Over long periods, compounding creates **very large differences in wealth**.

Key term definitions

- **Simple interest** — interest earned only on the original amount.
- **Compound interest** — interest earned on principal plus past interest.

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8. Frequency of Compounding

Interest does not always compound once per year.

It may compound:

- annually
- semi-annually
- quarterly
- monthly
- daily

General future value formula with m compounding periods per year

$$FV = PV\left(1+\frac{r}{m}\right)^{mt}$$

where:

- r = quoted annual rate
- m = number of compounding periods per year
- t = number of years.

Key intuition

More frequent compounding means:

- interest is added more often,
- so future value is slightly higher.

Key term definitions

- **Compounding frequency** — how often interest is added.
- **Periodic rate** — annual rate divided by number of compounding periods.

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9. Effective Annual Rate (EAR)

What is EAR?

The **Effective Annual Rate (EAR)** is the true annual growth rate after accounting for compounding.

Formula

$$EAR = \left(1 + \frac{r}{m}\right)^m - 1$$

Why it matters

Different quoted rates can be misleading unless they are expressed on the same effective basis.

Example logic

An interest rate of **8% p.a. compounded semi-annually** is not exactly 8% in effective terms:

$$EAR = \left(1 + \frac{0.08}{2}\right)^2 - 1 = 8.16\%$$

Key finance lesson

When comparing borrowing or investment options, **EAR is usually the fairest comparison tool**.

Key term definitions

- **Effective Annual Rate (EAR)** — actual annual rate after compounding.

- **Quoted / nominal rate** — stated annual rate before compounding adjustments.

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10. Annual Percentage Rate (APR) / Nominal Rates

A quoted annual rate is often a **nominal rate**, meaning it does not fully show the effect of compounding.

Key distinction

- **APR / nominal rate** tells you the stated annual rate.
- **EAR** tells you the actual annual growth rate.

Why students often confuse these

Two loans may both advertise “10%,” but if one compounds monthly and the other annually, the true cost differs.

Key term definitions

- **APR (Annual Percentage Rate)** — quoted annual borrowing rate, often before compounding adjustment.
- **Nominal rate** — stated annual rate not yet converted into an effective rate.

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11. Solving for Time, Rate, or Unknown Values

TVM formulas can be rearranged to solve for different unknowns.

You may be asked to solve for:

- the **future value**,
- the **present value**,
- the **interest rate**,
- or the **number of periods**.

Common rearrangements

From:

$$FV = PV(1+r)^n$$

You can solve for:

Interest rate

$$r = \left(\frac{FV}{PV}\right)^{1/n} - 1$$

Number of periods

$$n = \frac{\ln(FV/PV)}{\ln(1+r)}$$

Why this matters

These are practical finance questions such as:

- How long until an investment doubles?
- What return is implied by a price change?
- What rate is embedded in a contract?

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12. Annuities

What is an annuity?

An **annuity** is a series of equal cash flows paid at regular intervals for a fixed number of periods.

Examples

- loan repayments
- lease payments
- regular deposits into a savings plan

Two common annuity questions

1. What is the **present value** of those payments?
2. What is the **future value** of those payments?

Key term definitions

- **Annuity** — equal periodic cash flow stream for a finite number of periods.
- **Level cash flow** — same amount each period.

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13. Present Value of an Ordinary Annuity

An **ordinary annuity** pays at the **end** of each period.

Formula

$$PV_{annuity} = C \times \left(\frac{1-(1+r)^{-n}}{r} \right)$$

where:

- C = payment each period
- r = interest rate per period
- n = number of payments.

Why it matters

This formula is heavily used in:

- **loan valuation**,
- **mortgage repayments**,
- **bond coupon valuation**.

Key term definitions

- **Ordinary annuity** — annuity with payments at the end of each period.
- **Annuity factor** — bracketed term used to value the stream.

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14. Future Value of an Ordinary Annuity

Formula

$$FV_{annuity} = C \times \left(\frac{(1+r)^{n}-1}{r} \right)$$

Interpretation

This tells you how much a sequence of equal deposits will accumulate to by a future date.

Applications

Useful for:

- savings plans,
- superannuation / retirement contributions,

- sinking funds.

Key term definitions

- **Accumulated value** — total future amount built up from repeated deposits.
- **Sinking fund** — regular saving plan to reach a target future amount.

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15. Annuity Due

What is an annuity due?

An **annuity due** is an annuity where payments occur at the **beginning** of each period instead of the end.

Key implication

Because each payment occurs **one period earlier**, an annuity due is worth **more** than an otherwise identical ordinary annuity.

Adjustment rule

$$PV_{annuity\ due} = PV_{ordinary\ annuity}(1+r)$$

$$FV_{annuity\ due} = FV_{ordinary\ annuity}(1+r)$$

Key term definitions

- **Annuity due** — annuity with payments at the beginning of each period.

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16. Perpetuities

What is a perpetuity?

A **perpetuity** is a constant cash flow that continues **forever**.

Present value formula

$$PV_{perpetuity} = \frac{C}{r}$$

where:

- C = periodic cash flow
- r = required return per period.

Why perpetuities matter

Even though "forever" sounds unrealistic, perpetuities are important because they help value:

- some financial securities,
- preferred shares,
- simplified valuation models.

Key term definitions

- **Perpetuity** — equal cash flow received forever.

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17. Growing Cash Flows

Some cash flow streams are not level — they **grow over time**.

Growing perpetuity formula

$PV = \frac{C_1}{r-g}$

where:

- C_1 = next period's cash flow
- r = required return
- g = constant growth rate.

Important condition

This only works if:

$r > g$

Why it matters

This becomes especially important later for **share valuation** using dividends.

Key term definitions

- **Growth rate (g)** — expected rate at which cash flows increase over time.
- **Growing perpetuity** — perpetuity with cash flows that grow at a constant rate.

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18. Applications of TVM in Finance

Week 2 is not just maths — it is the basis for finance decision-making.

TVM is used to value:

- **loans**
- **bonds**
- **shares**
- **projects**
- **savings plans**
- **retirement contributions**

Core finance principle

A financial asset is generally worth:

> **the present value of the cash flows it is expected to generate**

This single idea links Week 2 to almost every later topic in the course.

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19. Common Mistakes to Avoid

1. Mixing annual and periodic rates

Always match:

- rate per period, and
- number of periods.

2. Ignoring timing

Know whether cash flows happen:

- today,
- at the end of the period,
- or at the beginning of the period.

3. Comparing values at different dates

You must move all values to the **same point in time** before comparing them.

4. Confusing APR and EAR

Nominal rates can hide the true cost or return when compounding differs.

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20. Week 2 Key Takeaways

- A dollar today is worth more than a dollar tomorrow.
- **Future value** moves money **forward** in time.
- **Present value** moves money **backward** in time.
- **Compounding** causes money to grow over time.
- **Discounting** is the foundation of valuation.
- More frequent compounding increases the effective return.
- **EAR** is better than quoted rates for fair comparison.
- **Annuities** are equal cash flow streams for a fixed number of periods.
- **Perpetuities** are constant cash flows that continue forever.
- Most finance problems are solved by valuing future cash flows at the **correct date**.

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21. Exam / Tutorial Focus Areas

Be confident with:

- calculating **future value** and **present value**
- distinguishing **simple vs compound interest**
- converting between **APR / nominal rates** and **EAR**
- working with **different compounding frequencies**
- valuing **multiple cash flows**
- valuing **ordinary annuities** and **annuities due**
- valuing **perpetuities**
- using **timelines** correctly
- rearranging formulas to solve for **rate** or **time**

Worked Examples

Example 1 — Future value of a single cash flow

Given: invest $5,000 at 7% p.a. for 10 years.

Use $FV_n = PV(1+r)^n$

- FV = 5,000 × $(1.07)^{10}$ = 5,000 × 1.96715 = **$9,835.76**

Example 2 — Present value of an ordinary annuity

Given: receive $2,000 at the end of each year for 5 years; r = 8%.

Use $PV = C \times \frac{1-(1+r)^{-n}}{r}$

- annuity factor = (1 − 1.08^{-5})/0.08 = (1 − 0.68058)/0.08 = 0.31942/0.08 = 3.99271
- PV = 2,000 × 3.99271 = **$7,985.42**

Example 3 — Future value of an ordinary annuity (savings plan)

Given: deposit $3,000 at the end of each year for 10 years; r = 6%.

Use $FV = C \times \frac{(1+r)^n - 1}{r}$

- factor = (1.06^{10} − 1)/0.06 = (1.79085 − 1)/0.06 = 0.79085/0.06 = 13.18079
- FV = 3,000 × 13.18079 = **$39,542.37**

Example 4 — Annuity due (payments at the start)

Given: the same stream as Example 2 ($2,000 for 5 years at 8%) but paid at the **beginning** of each period.

Use $PV_{due} = PV_{ordinary} \times (1+r)$

- PV_due = 7,985.42 × 1.08 = **$8,624.25**

It's worth more than the ordinary annuity because every payment arrives one period earlier.

Example 5 — Perpetuity and growing perpetuity

Given: a cash flow of $500 per year.

- **Level perpetuity** at r = 5%: $PV = \frac{C}{r}$ = 500 / 0.05 = **$10,000**

- **Growing perpetuity** (next payment $500, growth g = 2%, r = 5%): $PV = \frac{C_1}{r-g}$ = 500 / (0.05 − 0.02) = 500 / 0.03 = **$16,666.67**

Growth raises the value because each future payment is larger.

Example 6 — Solving for time (how long to double?)

Given: how many years for money to double at 8%?

Use $n = \frac{\ln(FV/PV)}{\ln(1+r)}$

- n = ln(2) / ln(1.08) = 0.6931 / 0.07696 = ≈ **9.0 years**

Formula Summary

Single cash flow

$FV_n = PV(1+r)^n$

$PV = \frac{FV_n}{(1+r)^n}$

Simple interest

$FV = PV(1+rn)$

Compounding more than once per year

$FV = PV\left(1+\frac{r}{m}\right)^{mt}$

Effective annual rate

$EAR = \left(1+\frac{r}{m}\right)^m - 1$

Solve for interest rate

$r = \left(\frac{FV}{PV}\right)^{1/n} - 1$

Solve for time

$n = \frac{\ln(FV/PV)}{\ln(1+r)}$

Present value of multiple cash flows

$PV = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \cdots$

Present value of ordinary annuity

$$PV_{annuity} = C \times \left(\frac{1-(1+r)^{-n}}{r}\right)$$

Future value of ordinary annuity

$$FV_{annuity} = C \times \left(\frac{(1+r)^{n}-1}{r}\right)$$

Annuity due adjustments

$$PV_{annuity\ due} = PV_{ordinary\ annuity}(1+r)$$

$$FV_{annuity\ due} = FV_{ordinary\ annuity}(1+r)$$

Perpetuity

$$PV_{perpetuity} = \frac{C}{r}$$

Growing perpetuity

$$PV = \frac{C_1}{r-g}$$

Readings

- Lecture notes are the primary source for this week
- This week underpins later topics in **debt**, **equity**, and **asset valuation**