Investments (FNCE30001)

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Topic 1: Zero-coupon bonds and term structure

Bond Fundamentals

- **Bonds** are issued by borrowers to a lender for an amount of cash. The issuer usually agrees to make specified payments (typically semi-annual, based on coupon rate) and repay the *face/par value* at the maturity date. All such details are listed in the bond indenture.
    1. While coupon rates are usually fixed, they can change depending on economic conditions (floating rate).
- **Zero-coupon bonds** make no coupon payments, and investors receive the par value at maturity. These bonds are priced considerably below par value, and the investor's return comes solely from the difference between par value and issue price.
- *overall, there have been an increasing number of bonds on issue, both government and non-government.

Pricing a default-free zero-coupon bond

Consider a default-free zero that pays $*Par* in *T* years:

$$P_0 = \frac{Par}{(1 + z_{0T})^T}$$

where z_{0T} is the interest rate that applies per year from time 0 to time T
z_{0T} = 'T-year zero coupon rate'

Regardless of term, zero rates are:

- Calculated on a compound interest basis
- Quoted per annum

We could also: use price → zero-coupon rate:

$$z_{0T} = \left(\frac{Par}{P_0}\right)^{\frac{1}{T}} - 1$$

- When the term to maturity (T) is larger, the bond price is more sensitive to changes in the interest rate.
- A *zero* is particularly sensitive because there are no interim payments and if a default occurs, investors lose all their value.
- Bond price = amount paid today to buy the bond
- Bond par value= amount to be received at maturity

Factors that affect zero-coupon bond prices

An increase in:	Bond price	Why?
Time to maturity	↓	Discounted more heavily as time is longer
Default risk of borrower	↓	Higher risk → Higher required return →lower price
Tax	↓	Higher required return → lower price, as after-tax returns need to be higher. Bonds also have tax incentives
Liquidity in the secondary market	↑	Reduced selling risk → lower required return →higher price.
Expected inflation	↓	Bond par value is worth less → lower price today

Australian money market securities

- A **money market security** is a short-term debt (<1 year) and is quoted using simple interest.

  The general formula for pricing money market securities in Australia is

  $$P_0 = \frac{Par}{1 + s \cdot \frac{n}{365}}$$

  where

  - s is the simple annual interest rate (yield)
  - *n* is the number of days until maturity
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Rates Terminology

- **Yield to Maturity** (YTM) is the discount rate that equates the price of the bond to the present value of the bond's future cash flows.
  1. In other words, the bond's internal rate of return given its current price.
- **Spot-rate** is the rate that we can borrow or lend at right now ('on the spot', same as current interest rate).
- A zero-coupon bond and the risk-free spot rate both represent borrowing or lending from time 0 to time T with a single payment at maturity. Since

they provide identical cash-flow structures and risk, no-arbitrage implies that their rates must be equal: *current zero rate (z0) = spot rate (r0)*.

Arbitrage (RISK FREE PROFIT)

Arbitrage = Simultaneous buying and (short) selling of equivalent securities, resulting in either

- no liabilities (no money paid out-of-pocket) today and positive cash flow in the future,
- positive cash flow today and no liabilities in the future, or
- positive cash flow today and positive cash flow in the future

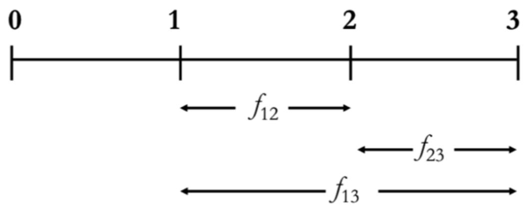
Sub-digression: What is short selling?

- Borrowing a security, selling it, and repurchasing it at a later date

- If an arbitrage opportunity exists (interest rate < zero rate), everyone will try to borrow money and buy a bond with it. The markets have a way of eliminating this:
    1. As more people borrow → banks increase interest rate
    2. As more people buy bond → price increases (zero rate decreases).

Forward Rates

- A forward contract is an agreement today to trade an asset at a fixed price on a fixed date in the future.
- **Forward rate (f)** is the interest rate set today (t0) for the period from one future date to another future date.



- If I want to invest $X for 5 years, I can either
    1. (1) buy $X worth of a 5 yr zero
    2. (2) buy $X worth of a 3yr zero and buy a forward contract for a 2 year zero (at t=3).
- No-arbitrage pricing would mean that (1) and (2) should result in the same returns

$$f_{tT} = \left(\frac{(1+z_{0T})^T}{(1+z_{0t})^t}\right)^{\frac{1}{T-t}} - 1$$

- Key idea: Forward rates ↔ Zero rates
- Equivalently, we may rewrite Equation 15.4 as

  $$(1+y_n)^n = (1+y_{n-1})^{n-1}(1+f_n)$$

The term structure of interest rates

- TSIR: the different interest rates that apply to different investment terms at a given time. This can be represented by commonly used curves: zero rate curves (z), forward rate curves (f, for one period investments) and yield curves (y).

Term structure hypotheses: We generally interpret the structure through one of 2 ways:

- **The expectations hypothesis**: the market sets today's term structure of zero rates such that the resulting forward rates are equal to the market's expectations of future market rates.
  1. It assumes that
     - Investors are risk neutral
     - There is no default risk

Consider the following ways to invest $X for 2 years:

1. Buy a 2-year zero (z_{02})
2. Buy a 1-year zero (z_{01})
   In one year, re-invest the proceeds at the prevailing 1-year interest rate (r_{12})
3. Buy a 1-year zero (z_{01}) and enter a 1-year forward contract starting in 1 year (f_{12})
   In one year, invest the proceeds at the 1-year forward rate

2 is risky, but 1 and 3 are not

In other words, the future rate is the expected market rate at that time.

2. The main weakness of the expectations hypothesis is that it assumes investors are risk-neutral (as opposed to risk-averse, which is the standard assumption [higher risk → higher expected return]).

We know that:

$$(1 + z_{0T})^T = (1 + z_{0t})^t (1 + f_{tT})^{T-t}$$

According to the Expectations Hypothesis:

$$(1 + z_{0T})^T = (1 + z_{0t})^t (1 + E[\tilde{r}_{tT}])^{T-t}$$

Implying:

$$f_{tT} = E[\tilde{r}_{tT}]$$

- **The liquidity premium hypothesis** has the key idea that the longer you lend, the more risk you incur as a lender, and therefore demand higher compensation.
  1. It assumes that lenders want to lend for shorter periods than borrowers want to borrow.
  2. Holding this assumption true, the difference between lending for one year and two years is caused by:
     - The expectation of the interest rate one year from today (E[r12])
     - A **premium** that must be paid to the lender for investing for a longer period (and sacrificing their liquidity).

$$(1 + z_{02})^2 = (1 + z_{01}) \cdot (1 + E[\tilde{r}_{12}] + \boldsymbol{L_{12}})$$

  3. $$f_{12} = E[\tilde{r}_{12}] + L_{12}$$
     - The return after 2 years is the zero rate in year 1, the expected market rate in year 2 PLUS a premium for investing for a longer period. This also implies that the forward rate reflects not only the expectations but the premium too.
  4. If a liquidity premium exists, long-term interest rates are higher than they would be if based purely on expectations of future interest rates.

- Both hypotheses express the importance of expectations.

Types of Yield Curves

- **Yield curves** plot interest rates (x-axis) against time to maturity (y-axis).
- **Normal yield curve** (upward sloping)
  1. Short-term rates are lower than long-term rates
  2. Exp Hyp → future rates will be higher
  3. Liq Hyp → compensation for interest rate risk (long-term investment)
- **Inverted yield curve** (downward sloping)
  1. Long- term rates are lower than short-term rates
  2. Exp Hyp and Liq hyp → future rates lower
  3. Interpretation: Recession is coming and investors are getting into long-term assets before rates drop.
- Arguments for inverted yield curve:
  1. Flight to higher quality investments
     - If investors sense instability, they may increase their purchases of long- term interest-bearing securities
     - High demand to lock in higher interest rates puts demand pressure on the prices of long-term debt, pushing up prices and yields down
  2. Investors expect lower interest rates in the future
     - Either because a weak economy reduces demand for capital
     - Or because interest rates are cut during recessions
- The yield curve provides info on the risk-free rates at different maturities, helps forecast recessions and assists bond trading strategies.

Topic 2: Coupon bonds and risky bonds

- Equivalent annual yield/rate: $EAY = \left(1 + \frac{APR}{k}\right)^k - 1$
- Coupon bond formula:

$$P_0 = \sum_{n=1}^{N} \frac{C}{(1+YTM)^n} + \frac{FV}{(1+YTM)^N}$$

- Practical issues with bond pricing formula. It assumes:
  1. Annual coupons: *most bonds pay half-yearly*