Solving for unknown interest rate

$$PV_0 = \frac{FV_n}{(1+r)^n}$$

When solving for r

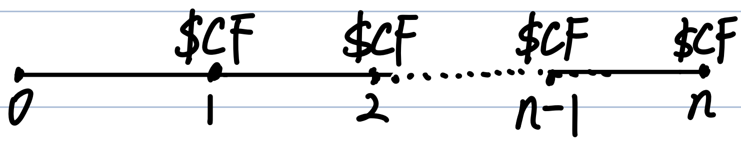
$$r = \left(\frac{FV_n}{PV_0}\right)^{1/n} - 1$$

When solving for n

$$n = \frac{\log\left(\frac{FV_n}{PV_0}\right)}{\log(1+r)}$$

Ordinary annuities

— An ordinary annuity is a series of equal, periodic cash flows, CF, occurring at the end of each period and lasting for n periods



Valuation (1) Present value

$$PV_0 = \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \frac{CF}{(1+r)^3} + \cdots \frac{CF}{(1+r)^n}$$

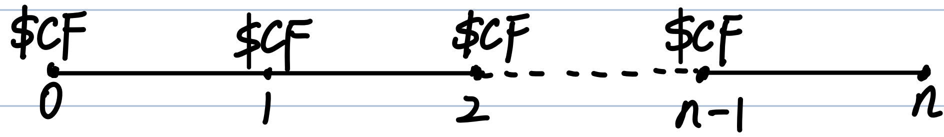
$$= \frac{CF}{r}\left[1 - \frac{1}{(1+r)^n}\right]$$

Valuation (2) Future value

$$FV_n = \frac{CF}{r}\left[1 - \frac{1}{(1+r)^n}\right] \times (1+r)^n$$
$$= PV_0 (1+r)^n$$

Annuities Due

An annuity due simply shifts all of the cash flows one period earlier.



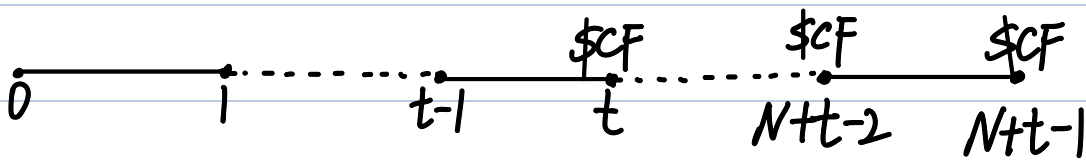
Value of an annuity due

$$PV_0 = \frac{CF}{r}\left[1 - \frac{1}{(1+r)^n}\right] \times (1+r)^1$$
$$= CF + \frac{CF}{r}\left[1 - \frac{1}{(1+r)^{n-1}}\right]$$
$$FV = PV_0 (1+r)^n$$

Deferred Annuities

Just like an ordinary annuity except it has a delayed start

Intuitively, it must be worth less than a standard ordinary annuity



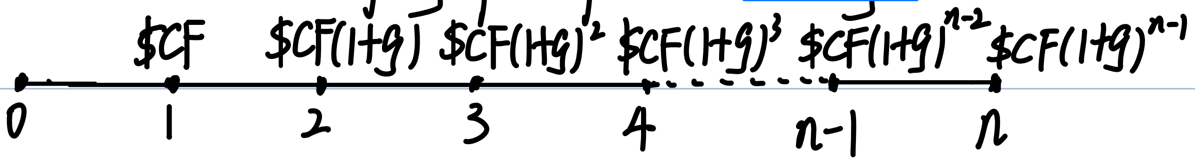
t — the number of periods until the first cash flow is paid

N — the number of cash flows paid

$$PV_0 = \left(\frac{CF}{r}\left[1 - \frac{1}{(1+r)^N}\right]\right) \times \frac{1}{(1+r)^{t-1}}$$

Constant Growth Annuities

Just like an ordinary annuity except the cash flows grow at a constant rate of g per period ($r>g$)

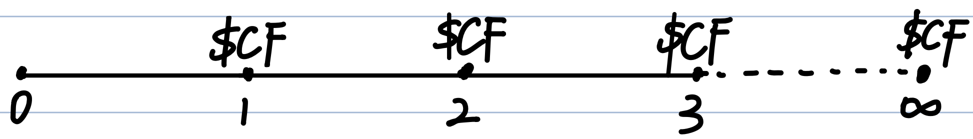


$$PV = \frac{CF}{r-g}\left[1 - \frac{(1+g)^n}{(1+r)^n}\right]$$

$$FV_n = \frac{CF}{r-g}\left[1 - \frac{(1+g)^n}{(1+r)^n}\right] \times (1+r)^n$$

Working with perpetuities

A perpetuity is an annuity which goes on infinitely



$$PV_0 = \frac{CF}{r}\left[1 - \frac{1}{(1+r)^\infty}\right] = \frac{CF}{r}$$

→ cash flow

= "Yield" = r

Different perpetuities

Deferred perpetuities

$$PV_0 = \frac{CF}{r} \times \frac{1}{(1+r)^{t-1}}$$

Perpetuities due

CF

$$PV_0 = CF + \frac{CF}{r}$$

Constant growth perpetuities ($r > g$)

A perpetuity where the regular cash flows are expected to grow at a constant rate g per period after the first cash flow (CF_1)

$$PV_0 = \frac{CF_1}{r-g}$$