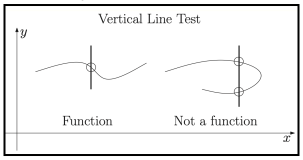
SCIE1500 Exam Notes

Functions:

Definition: y is a function of x if each value of x gives only one value of y.



Notation:

Closed interval: $x \in [a, b] \Rightarrow a \leq x \leq b$.
Open interval: $x \in (a, b) \Rightarrow a < x < b$.
Semi-open interval: $x \in (a, b] \Rightarrow a < x \leq b$.

1. $f(x) = x^2$ has domain $D = \{x : x \in \mathbb{R}\}$, range $R = \{y : y \in \mathbb{R}\ y \geq 0\}$.
2. $f(x) = \sqrt{x}$, $D = \{x : x \geq 0, x \in R\}$, $R = \{y \in \mathbb{R} : y \geq 0\}$.
3. $f(x) = 2^x$, $D = \{x : x \in \mathbb{R}\}$, $R = \{y \in \mathbb{R} : y > 0\}$.

Linear Functions: general form y= mx + c

Domain $D = \{x : x \in \mathbb{R}\}$, Range $R = \{y : y \in \mathbb{R}\}$

Quadratic Function: basic form y= x^2

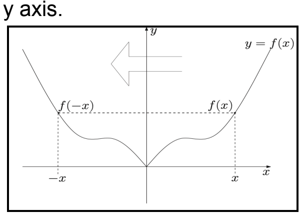
$$D = \{x : x \in \mathbb{R}\}, \quad \text{Range } R = \{y \in \mathbb{R} : y \geq 0\}$$

Sketching a quadratic:

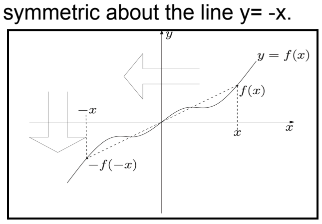
$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Find x intercepts if any: put y=0 and solve for x.
2. Find y intercept: put x=0 and solve for y.
3. Find the vertex: use the quadratic formula to find the x value. Substitute it to find y value.

Symmetry:
→ **Even function:** f(x) = f(-x); function is symmetric about the y axis.



→ **Odd symmetry:** f(-x) = -f(x); the graph of this function is symmetric about the line y= -x.



Power functions:

$$f(x) = x^a, a \neq 0,$$

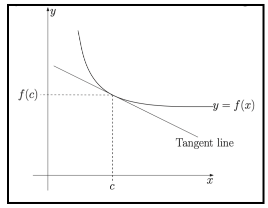
Exponential functions:

$$f(x) = a^x, a > 0$$

→ Find the y intercept by making x=0.

Differentiation:

Gradient:



$$\frac{f(x+h) - f(x)}{(x+h) - h} = \frac{f(x+h) - f(x)}{h}$$

→ y=mx+c → gradient is constant (linear line).
→ y=ax^2 + bx + c → gradient changes with x (quadratic).

Limits:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x+2)(x-2)}{(x-2)} = \lim_{x \to 2} (x+2) = 4$$

1.Factorise the numerator.
2.Cancel of the common term.
3.Substitute in x=2.
4.Answer = 4.
5.If the denominator does not cancel out then <u>the limit does not exist.</u>

OR:

$$\lim_{x \to \infty} \frac{x^2 + 2x + 1}{2x^2 - x - 4}$$

$$\lim_{x \to \infty} \frac{x^2 + 2x + 1}{2x^2 - x - 4} = \lim_{x \to \infty} \frac{1 + 2/x + 1/x^2}{2 - 1/x - 4/x^2}$$

$$= \frac{1}{2}$$

1.Identify the largest power of x in the expression (x^2).
2.Divide each term by this power.
3.Terms like x are then comparably so small they are effectively 0 so can be ignored so can cancel out to get ½.

Derivatives:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Product rule:

$$(f.g)' = f'.g + f.g'$$

$$f(x) = -e^{-x}(1 + x)$$

$$u(x) = -e^{-x} \Rightarrow u'(x) = -(-e^{-x}) = e^{-x}$$
$$v(x) = 1 + x \Rightarrow v'(x) = 1$$

$$f'(x) = e^{-x}(1 + x) + (-e^{-x})(1)$$
$$f'(x) = e^{-x}(1 + x) - e^{-x}$$
$$f'(x) = e^{-x}(1 + x - 1) = e^{-x}(x)$$

$$f'(x) = xe^{-x}$$

Quotient rule:

$$\left(\frac{f}{g}\right)' = \frac{f'.g - f.g'}{g^2}$$