Week 7 Risk And Return:

Historical Returns and Volatility:

One Period Realised Return:

- $R_{t+1} = \frac{D_{t+1}+P_{t+1}-P_t}{P_t} = \frac{D_{t+1}}{P_t} + \frac{P_{t+1}-P_t}{P_t}$
- $R_{t+1} = \frac{D_{t+1}+P_{t+1}-P_t}{P_t} = \frac{D_{t+1}}{P_t} + g$
- $R_{t+1} = Dividend\ Yield\ + Capital\ Gain$

When No dividends:

- $R_{t+1} = \frac{P_{t+1}-P_t}{P_t}$

Compounded Period Realised Return:

- $R_n = (1+R_1)*(1+R_2)\ldots*(1+R_t)-1$
    - Product of 1 + each periods return; then subtract 1.
- $CG_n = (1+CG_1)*\ldots*(1+CG_t)-1$
- $DY_n = (1+DY_1)*\ldots*(1+DY_t)-1$

Annualised Period Realised Return:

- One period holding return * n
    - n = periods in a year.
        - ➔ Annualized return from 6-month return: $R_{t+1} * 2$
        - ➔ Annualized dividend yield (DY) from 1-month return: DY*12

Arithmetic Historical Average Return:

- $\bar{R} = \frac{1}{T}(R_1 + R_2 + \cdots + R_T)$
- $E(R) = \bar{R}$
- Excel: AVERAGE ()

Geometric Average (better rep. of what happened)

- (1) 1+ Return (for all returns so all +)
- (2) Excel: GEOMEAN ()
- (3) (2) – 1 = geometric average return

Historical Variance and SD n

- $Var(R) = \frac{1}{T-1}\sum(R_i - \bar{R})^2$
    - Excel: VAR.S (Arthem. Returns)
- $\text{SD(R)} = \sqrt{\text{VAR(R)}}$
    - Excel: STDEV.S(Arthem. Returns)
      SD is measure of risk or volatility.

Interpreting Historical Volatility: PI

- $PI = \bar{R} \pm z * SD(R)$
    - $z = \frac{X-average}{SD}$
- 95% Returns fall between 2 SD of ER:
    - $\bar{R} \pm 2 * SD(R)$
- 67% Returns fall between 1 SD of ER:
    - $\bar{R} \pm 1 * SD(R)$

Future Returns and Volatility:

Single Asset Expected Return:

- $E(R) = \sum p_i * R_i$

Single Asset Expected Volatility:

- $\text{Var(R)} = \sum_{i=1}^{T} p_i * [R_i - E(R)]^2$
    - T = Total Number of States
    - Pi = Probability state (i) will occur
    - Ri = Expected Return if state (i) occurs
- $\text{SD(R)} = \sigma(\text{R}) = \sqrt{\text{VAR(R)}}$
    - *SD is the measure of risk or volatility*

Portfolio Weights:

- $Value_i = no.shares * Price$
- $Portfolio\ Value = \sum Value_i$
- $W_i = \frac{Value_i}{Portfolio\ Value}$

N Asset Portfolio Expected Return:

- $\text{E}(\text{R}_\text{p}) = \sum \text{w}_\text{j} * \text{E}(\text{R}_\text{j})$

Variance + SD of a 2 Asset Portfolio:

- $VAR(R_p) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2p_{1,2}w_1w_2\sigma_1\sigma_2$
- $SD(R_p) = \sqrt{Var(R_p)}$

Risk Diversification and Port Risk:

Portfolio Return: weighted average

- $E(R_A) \le E(R_p) \le E(R_B)$

Portfolio Volatility: not weighted average

- $SD(R_p) < weighted\ sum\ of\ risks$

Portfolio Total Risk:

- $\sigma^2_{Total} = \sigma^2_{Independent} + \sigma^2_{Common}$

Asset Pricing:

Ranking Portfolios: (higher PC = better portfolio).

- $Portfolio\ Compensation = \frac{Portfolio\ return}{Portfolio\ risk}$
    - *Also called Reward to Risk Ratio*

Market Portfolio: MM has highest RR Ratio

- $\sigma^2_{Total} = \sigma^2_{Independent} + \sigma^2_{MARKET\ RISK}$
    - $\sigma^2_{Total} = \sigma^2_{MARKET\ RISK}$

CAPM

Return Risk Compensation for Market Portfolio:

- $\frac{\text{Market portfolio risk premium}}{\text{Market portfolio risk}} = \frac{(\text{E}(\text{R}_\text{m})-\text{R}_\text{f})}{\sigma^2_{\text{Sys,M}}}$
    - $E(R_m) =$ $Weighted\ average\ of\ all\ risky\ assets$

Return Risk Compensation for Individual asset:

- $\frac{\text{Individual asset i risk premium}}{\text{Individual asset i systemtic risk}} = \frac{(\text{E}(\text{R}_\text{i})-\text{R}_\text{f})}{\sigma^2_{\text{Sys,i}}}$
    - $E(R_i) = Expected\ return\ for\ asset\ i$
    - $E(R_i) - R_f = RP\ for\ holding\ the\ SR\ of\ asset\ i.$