PSY248 Design and Statistics II

One-way ANOVA
An independent samples t-test only allows the experimenter to compare two levels of the independent variable (e.g. brown eyes vs blue eyes). Analysis of variance (ANOVA) is used to compare group means when there are multiple levels of the IV (e.g. brown eyes vs blue eyes vs green eyes).

Why not run multiple t-tests for every pair of groups?

- If, for example, we ran separate tests to compare three groups, we would have to carry out three tests: 1v2, 1v3 and 2v3.
- If each of these t-tests use a .05 level of significance, then the probability of falsely rejecting the null hypothesis (of making a Type 1 error) is only 5%. The probability of making *no* Type 1 errors is 0.95 for each test.
- Assuming that each test is independent, we can multiply the probabilities. The *overall* probability of no Type 1 errors becomes 0.95 x 0.95 x 0.95 = 0.857
- Therefore, the probability of making *at least* one Type 1 error across this group of tests becomes 1 - 0.857 = 0.143, or 14.3%
- The probability of making a Type 1 error has increased from 5% to 14.3% - we have lost control of the Type 1 error rate by running multiple t-tests.
- ANOVA allows us to compare multiple group means without losing control of the Type 1 error rate.

Model #1: single factor ANOVA with fixed effects, analysing a single IV between groups.

$$X_{ik} = \mu + \tau_k + \varepsilon_{ik}$$

- X – score
- μ – grand population mean. This is the arithmetic average of all scores.
- τ_k – effect parameter; the extent of difference between group means.
- ε_{ik} – error term; the extent to which scores within a group differ from each other.

With these definitions in mind, it follows that:

$$\tau_k = \mu_k - \mu$$

Effect parameter – difference between a group mean and the grand mean

$$\varepsilon_{ik} = X_{ik} - \mu_k$$

Error term – difference between a particular score and the mean of the group to which it belongs.

Thus we can rewrite the ANOVA model:

$$X_{ik} - \mu = (\mu_k - \mu) + (X_{ik} - \mu_k)$$

In this model, all terms are expressed as deviations. We want to know if a particular score in a group has the same value as the grand population mean. The total variation from the grand mean is divided into two terms: the effect parameter explains the variation **between** groups (model variation), while the error term explains the variation **within** groups (residual variation).

However, we usually do not have access to population parameters such as μ. Instead, we access sample statistics such as sample mean $\bar{X}$; hence the model is rewritten as such:

$$X_{ik} - \bar{X} = (\bar{X}_k - \bar{X}) + (X_{ik} - \bar{X}_k)$$

This model describes the deviation of a single observation from the sample mean.

Sums of squares express the total deviation of the whole sample.

Total sum of squares (SS_T):

$$\Sigma_i \Sigma_k (X_{ik} - \bar{X})^2$$

This is the total variation between all scores, regardless of the experimental condition from which the scores come. Total degrees of freedom is N – 1.

Model sum of squares (SS_M):

$$\mathrm{n}\Sigma_k (\bar{X}_k - \bar{X})^2$$

Model sums of squares tell us how much of the total variation can be explained by the fact that different scores come from different groups. These are the variations due to experimental manipulation. Model degrees of freedom is k – 1, where k is the number of groups.

Residual sum of squares (SS_R):

$$\Sigma_i \Sigma_k (X_{ik} - \overline{X_k})^2$$

Residual sums of squares tell us how much of the total variation cannot be explained by the model. These variations are caused by extraneous factors, such as individual differences between subjects. Residual degrees of freedom is k(n – 1), where n is the number of scores in a group.

Sums of squares are additive. $SS_T = SS_M + SS_R$.
The calculation of these requires group means; however, this may be problematic where the means are non-integers, which may result in rounding errors. Calculational formulae for sums of squares don't involve means, presented below:

Total sum of squares (SS_T):

$$\Sigma X^2 - (\Sigma X)^2 / N$$

N – total sample size.

Model sum of squares (SS_M):

$$\Sigma {T_k}^2 / n - (\Sigma X)^2 / N$$

T – sum of scores in each group.
n – number of observations in each group.

Residual sum of squares (SS_R):

$$\Sigma X^2 - \Sigma {T_k}^2 / n$$