KH Model (Endo)

No SS only BGP (K Y k y grow at g* not constant)

$\boxed{Y = AK^{\alpha}H^{1-\alpha}}$ $Y = AK^{\alpha}H^{\beta}N^{1-\alpha-\beta}$ (β=1-α)

- CRS to accumulable factor (H and K)
- Diminishing return in K and H (only in SR not LR)

$\boxed{K_{t+1} - K_t = gK_t = s_K Y_t - \delta K_t}$ $\boxed{H_{t+1} - H_t = gH_t = s_H Y_t - \delta H_t}$

Intensity of H relative to K (H/K):

$\boxed{\frac{H_t}{K_t} = \frac{s_H}{s_K} \equiv \phi^*}$

$\rightarrow$ H > K: investment in H s_H > K s_K
- Constant in BGP= Φ*

SR Dynamics:

- $\phi_t \mapsto \phi_{t+1}$ intensity today = intensity next period

Initial H low: $\phi_0 < \phi^*$ $\frac{H_0}{K_0} < \phi^*$	Initial K low: $\phi_0 > \phi^*$ $\frac{H_0}{K_0} > \phi^*$
H accumulate faster > K $\rightarrow \phi_t$ rise over time : $\phi_{t+1} > \phi_t$	K accumulate faster > H $\rightarrow \phi_t$ fall over time: $\phi_{t+1} < \phi_t$
LR (BGP): converge to $\Phi^*=\Phi_t=\Phi_{t+1}=s_H/s_K$	

LR BGP:

$H_t = \phi^* K_t \rightarrow Y_t = A(\phi^*)^{1-\alpha}K_t$, $K_{t+1} - K_t = gK_t = s_K A(\phi^*)^{1-\alpha}K_t - \delta K_t$

- A*a constant*K: no diminishing returns to K H
- In BGP: K H constant proportion = Φ*

$\boxed{g^* = sA - \delta}$, $s \equiv s_K^{\alpha} s_H^{1-\alpha}$

- Diff s (bc diff s_K or s_H)= diff growth g*: $s_1 < s_2 \rightarrow g_2^* = s_2 A - \delta > g_1^*$
- Initial K_0 permanent effect on Y level

AK	KH
sustained growth bc no drt K K accumulable: K ↑ = Y ↑ = investment ↑ = K ↑ again	sustained growth bc Y CRS in accumulable factors H K K H growth = Y ↑ = investment increase = again K H growth

If $\boxed{Y = AK^{\alpha}H^{\beta}N^{1-\alpha-\beta}}$: N not accumulable (CRS in KHN but drt acc factors KH)

- No sustained growth bc more K H accumulation = DRS in Y

Growth Accounting

$\boxed{Y = AK^{\alpha}N^{1-\alpha}}$ $\qquad \ln Y = \ln A + \alpha \ln K + (1-\alpha)\ln N$

TFP: $A = \dfrac{Y}{K^{\alpha}N^{1-\alpha}} = \dfrac{Y}{N} \div (\dfrac{K}{N})^{\alpha}$: factor contribute to Y other than K N

$g_A = g_{\frac{Y}{N}} - \alpha g_{\frac{K}{N}}$

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Growth Rate Decomposition

$\boxed{g_Y = \dfrac{\ln Y_T - \ln Y_0}{T}}$ *same for A K N*

$\boxed{g_Y = g_A + \alpha g_K + (1-\alpha) g_N}$

$g_{Y/N}$ = $\boxed{g_Y - g_N = g_A + (1/3)(g_K - g_N)}$ *Contribution of $g_{K/N}$ include 1/3*

- $g_K - g_N$: capital deepening $\longrightarrow$ K/N grow if $g_K - g_N > 0$ (always use K/N when /N)

Level Decomposition (diff econ)

$\ln Y_i = \ln A_i + \alpha \ln K_i + (1-\alpha)\ln N_i$

$\ln Y_j = \ln A_j + \alpha \ln K_j + (1-\alpha)\ln N_j$

Differences in Y:

$$\boxed{\begin{aligned}\ln Y_i - \ln Y_j = \ln A_i - \ln A_j + \alpha\left(\ln K_i - \ln K_j\right) \\ + (1-\alpha)\left(\ln N_i - \ln N_j\right)\end{aligned}}$$

Differences in Y/N:

$$\boxed{\ln \frac{Y_i}{N_i} - \ln \frac{Y_j}{N_j} = \ln A_i - \ln A_j + \alpha\left(\ln \frac{K_i}{N_i} - \ln \frac{K_j}{N_j}\right)}$$ $\longrightarrow$ diff in TFP+diff in K/N

A contribution $\qquad$ K/N contribution

Calculation

- Find Y/N, K/N, A of two economies
- Find ratio of $\frac{Y_i}{N_i} / \frac{Y_j}{N_j}$ $\quad A_i/A_j$ $\quad \frac{K_i}{N_i} / \frac{K_j}{N_j}$
- Find log difference: $\ln\left(\frac{Y_i}{N_i} / \frac{Y_j}{N_j}\right)$ $\ln(A_i/A_j)$ $\ln\left(\frac{K_i}{N_i} / \frac{K_j}{N_j}\right)$ same as $\ln A_i - \ln A_j$ and $\left(\ln \frac{K_i}{N_i} - \ln \frac{K_j}{N_j}\right)$
- Find A $\ln A_i - \ln A_j$ and K/N $\alpha\left(\ln \frac{K_i}{N_i} - \ln \frac{K_j}{N_j}\right)$ contributions to $\ln \frac{Y_i}{N_i} - \ln \frac{Y_j}{N_j}$

TFP as residual

If $\ln Y = \ln A + \alpha \ln K + (1-\alpha)\ln(NH)$, $\ln \hat{A} = \ln A + (1-\alpha)\ln H$

- More things in prod function = smaller contribution of TFP (A) will be to Y

<u>Calculation – Monetary policy making</u>

- Loss function: $L(u, \pi) = u^2 + \pi^2, \qquad (\text{i.e., } \gamma = 1)$
- PC: $u = 0.05 - (\pi - \mathbb{E}(\pi)), \qquad (\text{i.e., } \alpha = 1,\ \overline{u} = 5\%)$
  - Sub PC u into L: $L = [0.05 - (\pi - \mathbb{E}(\pi))]^2 + \pi^2$
  - Min L by choose π: $\frac{dL}{d\pi} = 0$
    - $\frac{dL}{d\pi} = -2[0.05 - (\pi - \mathbb{E}(\pi))] + 2\pi = 0$
    - $\pi = 0.025 + 0.5\mathbb{E}(\pi) \longrightarrow$ f(E(π)) best response function given E(π)
- Equilibrium: $\pi = \mathbb{E}(\pi) = \pi^* \quad \Leftrightarrow \quad \pi^* = 0.025 + 0.5\pi^*$
  - $\pi^* = 0.05$ > 0% under Rule

$L^{Disc.} = \bar{u}^2 + \pi^{*2} \qquad L^{Rule} = \bar{u}^2 + \sigma^2$

- Loss higher in Discretion: $L^{Disc} = 0.05^2 + 0.05^2 > L^{Rule} = \bar{u}^2$