# Lecture 7 and 8
# Income and Substitution Effect

(continue from previous content)

- X not affected by price of y bc Cobb-Douglas function
- Explain why demand curve slope downward

- Substitution Effect $\Delta x^S$ : change in x due to relative price change (Px/Py) only
    - Ex. From 9/16 to 16/16 – new budget line steeper
    - Always in opposite (non-positive) direction of change in Px
        - $\frac{\Delta x^S}{\Delta p_x} \leq 0$
        - When X becomes more expensive relative to other good, consumer substitute out the expensive X into cheaper Y

- Income Effect $\Delta x^M$ : change in x due to change in real income (M/Px) only
    - Ex. real income in terms of x decreases when price of x increases (less x can buy using all M)
        - Original budget line (blue) no longer affordable
    - Real income changes in opposite direction of price change
        - $\frac{\Delta \, Real \, M}{\Delta p_x} < 0$
    - Income effect (Normal good)
        - Normal good: $\frac{\Delta x^M}{\Delta \, Real \, M} > 0$ Qd moves in same direction as income
        - Normal good: Qd moves in *opposite direction* as price (Px)
            - $\frac{\Delta x^M}{\Delta p_x} < 0$
        - $\frac{\Delta x}{\Delta p_x} = \underset{(-)}{\frac{\Delta x^S}{\Delta p_x}} + \underset{(-)}{\frac{\Delta x^M}{\Delta p_x}} < 0$
    - Income effect (Inferior good)
        - Inferior good: $\frac{\Delta x^M}{\Delta \text{ Real } M} < 0$ Qd moves in opposite direction as income
        - Inferior good: Qd moves in *ambiguous direction* as price (Px)

- $$\frac{\Delta x^M}{\Delta p_x} > 0$$
  - $$\frac{\Delta x}{\Delta p_x} = \frac{\Delta x^S}{\Delta p_x} + \frac{\Delta x^M}{\Delta p_x} \gtreqless 0$$
    (−) (+)

**Giffen Good**

- Qd increases as price increases
  - $$\frac{\Delta x}{\Delta p_x} > 0$$
  - So Giffen good must be an inferior good *and* positive income effect outweigh sub effect
    - $$\Delta x = \Delta x^S + \Delta x^M$$
    - $$|\Delta x^M| > |\Delta x^S|$$
- Giffen good is very *rare*:
  - The larger the consumption of this good is = larger income effect
    - Larger part of budget: they are mostly normal good (ex. house, food)
  - So person has to be poor to have an inferior good occupying large budget
    - Ex. although price of bread increases, still cheaper than meat, poor ppl still consume though increased price

**Application of Sub and Income Effect**

- For all good: normal and inferior with higher sub effect has *not upward* demand curve
  - Qd decrease as P increase
  - Inferior with higher sub effect: sub move Qd positively and income move Qd negatively
- Application: tax petrol to reduce carbon:
  - must know income effect to rebate people to hold real income constant
  - must know sub effect to know how much carbon will change
- Other situations need income effect
  - Change in pension/welfare/unemployment benefits due to *uneven inflation*

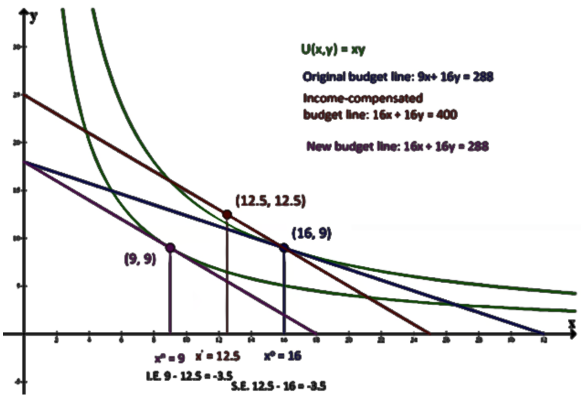
**Separating 2 Effects**

- To know sub effect: hold real income constant
  - Compensate income M to keep real income constant at new prices
    - orange line
  - Then parallel shift (orange) that to intercepts with actual M (pink)
  - Distance (negative) from blue to orange in △x = sub effect
  - Distance (negative) from orange to pink in △x = income effect

## Method 1: Slutsky Method

- Constant real income: give consumer just enough income to afford original bundle ($x^0$, $y^0$)
  - But will not consume original bundle bc (x', y') above that IC of blue curve: more preferred (x', y') > ($x^0$, $y^0$)

- **Calculation**: (x', y') is demand for x, y at *new price* if given income M' with *same real income* so can afford original bundle
  - $$M' = p_x^n x^o + p_y y^0$$
    - Ex. 16(new Px)*16(original x) + 16 (Py)*9(original y) = 400 (M')
    - Income-compensated BL: new Px*x + Py*Y = M'
  - How much to consume: sub to demand function with new M'
    - Get x' and y'
  - Shift income-compensated BL (orange) to new BL (pink)
    - new Px*x + Py*Y = M
  - Calculate sub and income effect
    - $x^n - x^o = x' - x^o + x^n - x'$
    - Income effect always between *parallel* lines (same relative P)




- **Analysis of Slutsky**
  - Advantage: data necessary to calculate M' are observable
    - $x^0$, $y^0$, new Px ($p^n_x$), Py
    - so more used in applied work: CPI calculation (measure inflation calculate diff in cost to buy same good)
  - Disadvantage: Slutsky doesn't hold utility constant
    - Real income: ability to buy products bc gives utility
      - so it's better to hold utility constant
      - (x', y') better than ($x^0$, $y^0$)
    - Less useful in theory

## Method 2: Hicks Method

- **Constant utility**
  - Parallel shift in *new* Budget Line to tangent to *old* IC (so still with same utility)
  - Give new M' so at new prices just as happy as at old bundle ($x^0$, $y^0$)
    - (x', y') to ($x^0$, $y^0$): all sub effect bc real income constant (in this case real income = utility same)
    - ($x^n$, $y^n$) to (x', y'): all income effect bc relative price constant

- diff to Slutsky: (x', y') on same IC
- known ($x^0$, $y^0$) and ($x^n$, $y^n$)

- **Calculation**:
  - Plug ($x^0$, $y^0$) into utility function to get U($x^0$, $y^0$)
  - (x', y') is Qd at new prices with new M' to maintain Utility same as U($x^0$, $y^0$)
    - U(x', y') = U($x^0$, $y^0$)
  - Substitute x' and y' demand functions new demand functions at new price (new Px/Py) (and with M') to U(x', y')
    - Ex.
      - $U(x', y') = x'y'$ and $x' = \frac{M'}{2p_x^n}$ and $y' = \frac{M'}{2p_y}$
      - $U(x', y') = x'y' = \frac{M'}{2p_x^n} \cdot \frac{M'}{2p_y} = \frac{M'^2}{4p_x^n p_y}$
  - Equate U(x', y') to value of U($x^0$, $y^0$)
    - Ex.
      - At new prices $(p_x^n, p_y) = (16, 16)$, $U(x', y') = \frac{M'^2}{1024}$
      - Set $U(x', y') = U(x^o, y^o)$: $\frac{M'^2}{1024} = 144$
      - Solve for M: $M' = 384$
  - Plug in M' into x' and y' demand functions get (x', y')
    - Ex.
      - $x' = \frac{M'}{2p_x^n} = \frac{384}{32} = 12$ and $y' = \frac{M'}{2p_y} = \frac{384}{32} = 12$
  - Calculate sub (x' – $x^0$) and income effect ($x^n$ – x')
    - Diff to Slutsky

- **Analysis of Hicks**
  - Disadvantage: utility function not observable so only useful in theory
  - Advantage: correct bias of Slutsky
    - Bc keep utility constant: M needed is less
      - CPI overcompensate ppl by overestimate inflation: if ppl actually have 400 budget: will buy at (12.5, 12.5) and has higher utility
      - At (12, 12) ppl will be truly same as before

## Method 3: Kaldor Method

- Constant utility: but constant utility to *new* Budget Line with *old* prices
- Give new M' so at old prices just as happy as at new bundle ($x^n$, $y^n$)
  - (x', y') to ($x^0$, $y^0$): all income effect bc prices same (relative price constant)
  - ($x^n$, $y^n$) to (x', y'): all sub effect bc real income constant (utility constant)

- Calculation: set U(x', y') = U($x^n$, $y^n$)