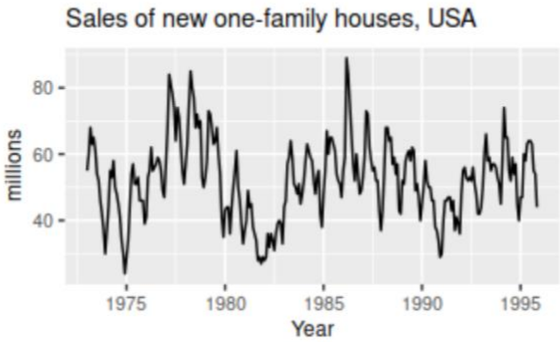
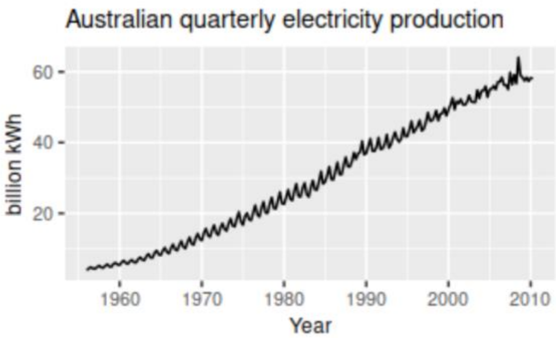
Forecasting Trends and Seasonality

- General features of time series can be classified within three categories
  - Trends
  - Seasonality
  - Cycles
- A time series usually exhibits two or more of these features
  - E.g. seasonal and cyclical



  - E.g. seasonal and trending



<u>Trends</u>

- Trend – a smooth, typically unidirectional pattern in the data that arises from the accumulation of information over time
  - i.e. a long-term increase or decrease in the data
  - Doesn't have to be linear
- ACF of trended series tend to have positive values that slowly decrease as the lags increase
  - i.e. When the data is trending ACF for small lags tend to be large and positive
  - This is because observations nearby are also nearby in size

Trend Models

- Trend models are (relatively) easy to fit and forecast
- 4 types
  - Linear – $y_t = \alpha + \beta t$
    - Simplest model that can be used to account for a trending time series

- Polynomial – $y_t = \alpha + \beta_1 t + \beta_2 t^2 + \cdots + \beta_p t^p$
  - E.g. quadratic, cubic, etc.
  - Caution is needed with (higher order) ones, as they may fit well in-sample, but this does not translate to good performance for out-of-sample
  - In fact, it is basically guaranteed that as you introduce more and more higher order polynomials, the out-of-sample forecast will deteriorate considerably
- Exponential – $y_t = e^{\alpha+\beta t}$ OR $\ln(y_t) = \alpha + \beta t$
  - Suitable when a time series is characterised with a stable relative/percentage change over time (e.g. GDP)
- Shifting (or switching) – $y_t = \alpha + \beta_1 t + \beta_2 (t - \tau) I(t > \tau),\ \tau \in T$
  - Within some range, there is a linear trend but then at some point there is a shift and we end up getting a different trend

- Exponential trend is same as a linear tend fitted to natural logarithm of the series
  - For a time series $\{y_t : t = 1, \ldots, T\}$, the natural log is $z_t = \ln(y_t)$
  - Transforming the model is beneficial as the transformed model
    - Is easier to interpret (can identify relative/percentage change)
    - Homogenizes the variance of the time series
    - May result in improved forecasting accuracy
  - Fitted trend can be reverse-transformed to fit the original series – $\hat{y}_t = e^{\hat{z}_t}$

Fitting and Forecasting Trends

- Generic representation of a trend model with an additive error term – $y_t = g(t;\theta) + \varepsilon_t$
  - Estimate θ by fitting the trend model to a time series using the least-squares regression – $\hat{\theta} = arg\min_{\theta} \sum_{t=1}^{T} (y_t - g(t;\theta))^2$
  - Fitted values are given by – $\hat{y}_t = g(t;\hat{\theta})$
- Any future *realization* of a random variable assumed to follow a linear trend model is $y_{t+h} = \alpha + \beta(t+h) + \varepsilon_{t+h}$
  - $t+h$ is the trend variable
- Optimal *forecast* – $y_{t+h|t} = E(y_{t+h}|\Omega_t) = E[\alpha + \beta(t+h) + \varepsilon_{t+h}] = \alpha + \beta(t+h)$
  - i.e. $E(y_t) = \alpha + \beta t$ – doesn't satisfy stationarity condition
  - Forecast error – $e_{t+h|t} = y_{t+h} - y_{t+h|t} = \varepsilon_{t+h}$
    - This implies that the forecast error from this model will have exactly the same characteristics as what we assume about ε (white noise)
  - Forecast variance – $\sigma^2_{t+h|t} = E\left(e^2_{t+h|t}\right) = E(\varepsilon^2_{t+h}) = \sigma^2, \quad \forall h$
    - i.e. $Var(y_t) = \sigma^2$ – variance is time-invariant
    - Assumes forecast variance doesn't change with the horizon, which is perhaps an incorrect assumption

- Density forecast – $f\left(Y_{t+h|t}\middle|\Omega_t\right) \rightarrow N(\alpha + \beta(t+h), \sigma^2)$
- Features of trend forecasts
  - Tend to understate uncertainty at long horizons
  - Short-term trend forecasts *can* perform poorly, while long-term trend forecasts *typically* perform poorly
  - Sometimes it may be beneficial to forecast growth rates and reconstruct level forecasts from growth