where;
β_{GDP} = Factor sensitivity for GDP
RP_{GDP} = Risk Premium for GDP
β_{IR} = Factor sensitivity for interest rate
RP_{IR} = Risk premium for interest rate

Arbitrage Pricing Theory

An **abitrage** opportunity arises when an investor can earn riskless profits without making a net investment. This is done by taking advantage of a price differential between two or more markets: the simultaneous purchase and sale of equivalent securities to profit from the imbalance in their prices.

Essence of the APT: A security's expected return and risk are directly related to its sensitivities to changes in one or more factors. The APT only holds exactly for well-diversified portfolios.

The law of one price states that if two assets are equivalent in all economically relevant respects, then they should have the same market price.

The critical property of a risk-free arbitrage portfolio is that any investor, regardless of risk aversion or wealth, will want to take an infinite position in it. When arbitrage opportunities exist each investor wants to take as large a position as possible; hence it will not take many investors to bring the price pressures necessary to restore equilibrium.

- Developed by Stephan Ross in 1976
- APT relies on three assumptions:
    1. Security returns can be described by a factor model
    2. There are sufficient securities to diversify away firm-specific risk
    3. Well-functioning security markets **do not allow** for the persistence of arbitrage opportunities
- In order to implement the APT we need to know what the factors are. Here the theory gives no guidance.

- There is some evidence that the following macroeconomic variables may be risk factors:
    - Major political upheavels
    - Changes in interest rates
    - Changes in monthly GDP
    - Changes in the default risk premium
    - Changes in expected inflation
    - Unexpected changes in the price level and many more...
- In the CAPM, only beta (measures exposure of a stock/portfolio to market-wide risk factors) is relevant

$$r_i = E(r_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$$

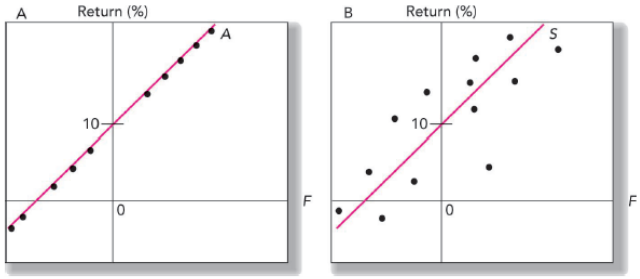
For i = 1 to N securities, where:

- r_i = return on asset i during a specified time period
- $E(r_i)$ = expected return for asset i
- β_{ik} = reaction in asset i's returns to movements in a common factor. Measures how each asset (i) reacts to a common factor (k)

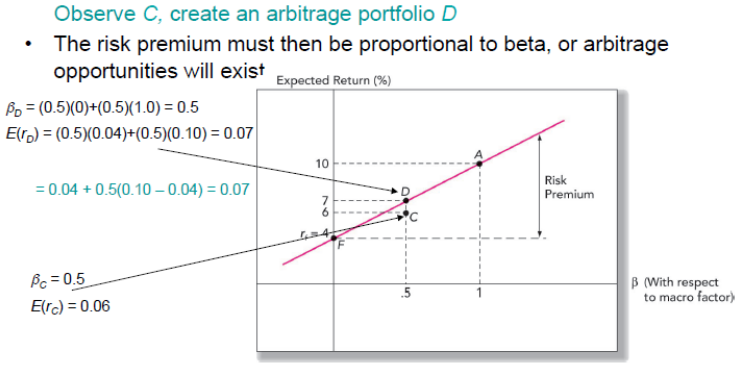
- F_k = a common factor with a zero mean that influences the returns on all assets
- e_i = a unique effect on asset i's return that, by assumption, is completely diversifiable in large portfolios and has a mean of zero
- N = number of assets

- Portfolio variance: $\sigma^2 = \beta_P^2\sigma_F^2 + \sigma^2(e_P)$
- Nonsystematic (firm specific) risk is given by: $\sigma^2(e_P) = \sum w_i^2\sigma^2(e_i)$
- If the portfolio is equally-weighted: $\sigma^2(e_P) = \frac{1}{n}\bar{\sigma}^2(e_i)$
  - Therefore, as the portfolio gets large, the portfolio's non-systematic variance approaches zero
  - We can assume: $r_P = E(r_P) + \beta_P F$

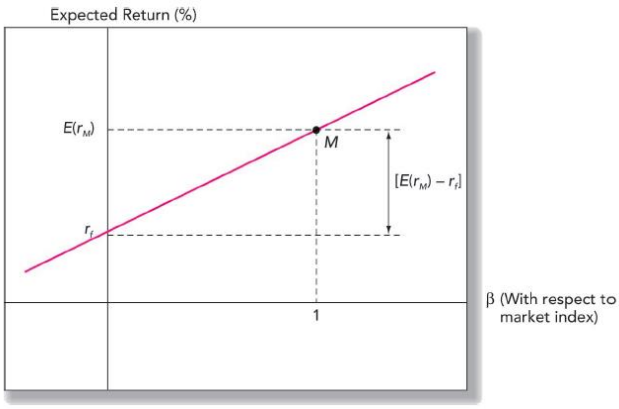
Beta and Expected Risk



- Since the non-systematic risk can be diversified away, investors should only be rewarded for the factor risk they hold
- Thus, the return of a well diversified portfolio A should only be determined by the systematic factor
- Assuming the expected return is 10% and beta is 1.0:
- If there is another well-diversified portfolio with an expected return of 8% and beta of 1.0, there will be an arbitrage opportunity
- Investors will short the second portfolio (B) and buy the first portfolio (A), thus making a riskless profit.
- Therefore, **the APT states that well-diversified portfolios with the same betas must have the same expected returns in the market equilibrium**



The Security Market Line



- The market index portfolio can be considered as a well-diversified portfolio
- The beta of the market index portfolio, by definition is 1.0

- Equation (equivalent to CAPM):

$$E(r_P) = r_f + [E(r_M) - r_f]\beta_P$$

Multifactor APT

$$r_i = E(r_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \beta_{i3}F_3 + \ldots..$$

- Requires formation of exactly priced factor portfolios to use as arbitrage portfolios
- A factor portfolio is a well-diversified portfolio that has a beta of 1.0 on one of the factors and a beta of 0 on any other factor

Example

Expected returns:

Factor portfolio 1: $E(r_{F1}) = 10\%$

Factor portfolio 2: $E(r_{F2}) = 12\%$

Risk-free rate: 4%

Portfolio C : $E(r_c) = 15\%$, $\beta_{C,F1} = 0.50$, $\beta_{C,F2} = 0.75$

Create an arbitrage portfolio

Portfolio A

Beta on factor portfolio 1: $\beta_{F1} = 0.50$

Beta on factor portfolio 2: $\beta_{F2} = 0.75$

$E(r_A) = 0.04 + (0.50)(0.10-0.04) + (0.75)(0.12-0.04) = 13\%$

Long C, Short A (50% of F1, 75% of F2 and -25% risk-free)

APT and CAPM Compared

- APT applies to well diversified portfolios but not necessarily to individual stocks
- With APT it is possible for some individual stocks to be mispriced - ie. not lie on the SML
- APT is more general than CAPM in that it gets to an expected return and beta relationship without the assumption of the market portfolio
- APT can be extended to multifactor models

Strengths of APT	**Weaknesses of APT**
The model gives a reasonable description of return and risk	Model itself does not say what the right factors are
Factors are plausible	Factors can change over time
No need to measure market portfolio correctly	Estimating multi-factor models requires more data

Summary

- Multifactor models seek to improve the explanatory power of single-factor models by explicitly accounting for the various systematic components of security risk. These models use indicators intended to capture a wide range of macroeconomic risk factors
- Once we allow for multiple risk factors, we conclude that the security market line also ought to be multidimensional, with exposure to each risk factor contributing to the total risk premium of the security
- A (risk-free) arbitrage opportunity arises when two or more security prices enable investors to construct a zero net investment portfolio that will yield a sure profit. The presence of