5. Stress-Strain behavior of soils

Sand behavior

- Usually sheared under drained conditions (relatively high permeability means excess pore pressures are not generated).
- Parameters governing sand behaviour is:
  - Relative density
  - Effective stress level

**Relative density:**

\[ I_d = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}} \]

Where \( e_{\text{max}} \) and \( e_{\text{min}} \) are the maximum and minimum void ratios.

Sand is generally referred to as dense if \( I_d > 0.6 \) and loose if \( I_d < 0.3 \).

Observations from the graphs below:

1. All samples approach the same ultimate conditions of shear stress and void ratio, irrespective of the initial density
2. Initially dense samples attain higher peak angles of friction \((\phi' = \tan^{-1}(\tau/\sigma'))\)
3. Initially dense soils expand (dilate) when sheared, and initially loose soils compress
4. The loose sand compresses until it reaches a plateau in volume strain
5. The dense sand expands, then the expanding begins to slow down before reaching an ultimate resistance.
6. The medium density sand begins by contracting and then starts to expand.
7. Note: it is possible for loose sand to expand!

Effective stress:

Observations from the graphs below:

1. The ultimate values of shear stress and void ratio, depend on the stress level, but the ultimate angle of friction \((\phi'_{\text{ult}} = \tan^{-1}(\tau/\sigma'))_{\text{ult}}\) is independent of both density and stress level
2. Initially dense samples attain higher peak angles of friction \((\phi' = \tan^{-1}(\tau/\sigma'))\), but the peak friction angle reduces as the stress level increases.
3. Initially dense soils expand (dilate) when sheared, and initially loose soils compress. Increasing stress level causes less dilation (greater compression).
\[ \tau = \tan \phi' \mu \]
- When soil is sheared it will eventually attain a unique stress ratio ($\frac{\tau}{\sigma'} = \tan \phi'_{ult}$) and reach a critical void ratio which is uniquely related to the normal stress.

The ultimate (ultimate) state equation:

$$\frac{d\tau}{d\gamma} = \frac{d\sigma'}{d\gamma} = \frac{d\varepsilon_v}{d\gamma} = 0$$

Soil is controlled by 3 things:

1. Shear stress
2. Void ratio
3. Effective stress

This equation produces a critical state line (CSL) which can be drawn in 2D representations for simplification:

![Diagram of critical state line](image)

In 3D the CSL looks like this:
Note:

- At critical states soil behaves as a purely frictional material
- Typical values for the ultimate value of $\phi$ are:
  - Clay: 22 degrees
  - Sand: 33 degrees
  - Gravel: 40 degrees

**Stress-Dilatancy relationship (sand)**

- Frictional resistance of soil may appear to be greater than $\phi'_{ult}$
- If $\phi' > \phi'_{ult}$ the soil is EXPANDING
- If $\phi' < \phi'_{ult}$ the soil is COMPRESSING
- If $\phi' = \phi'_{ult}$ the soil is deforming at a constant volume

Equation to remember:

$$\phi' = \phi'_{ult} + \alpha$$

Where $\alpha$ is the rate of volumetric strain

**Peak conditions (sand)**

$$\phi'_{pk} \quad I_d = 1$$
$$\phi'_{ult} \quad I_d = 0$$

$$I_d = 0.5$$
From the graph above it is evident that the peak angle cannot fall below the ultimate angle. 
I(d) = 1 is a dense sand, where as I(d) = 0 is a loose sand 
The horizontal axis is the log of the effective stress 
This graph means that loose sands may expand when sheared

Clay behavior
Essentially identical to the behaviour of sands 
Data is usually presented in terms of the soils stress history (OCR) rather than relative density 
Remember that: \( OCR = \frac{\sigma'_{pc}}{\sigma'} \)
The CSL is parallel to the normal consolidation line and lies below it in a void ratio vs. effective stress plot (shown below) 
We don’t know why its parallel and can’t explain why, we just know it is from observation

Normally consolidated clays behave similarly to loose sands 
Heavily over consolidated clays behave similarly to dense sands 
As the OCR increases (the soil gets denser) there is a gradual trend between the extremes:
In the first image, as the shear stress initially increases it does not actually hit the CSL (remember its actually meant to be 3D)

**Undrained response of clays**
- Volume change does not exist (prevented in tests)
- Therefore the **void ratio must be constant** (this is evident in the bottom left graph)
- Soil always heads towards a critical state when sheared
- If the final total stresses are known then the excess pore pressures can be determined

Understanding the CSL helps to explain the 'apparent cohesion' (undrained strength) in frictional materials
- If the moisture content changes then the undrained strength will change too
- This is because failure will occur at a different point on the CSL

**Differences between sands and clay**

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- Broad patterns of behaviour observed for sands and clays are very similar