- ➔ Bohr proposed that electron angular momentum (L) is quantised.

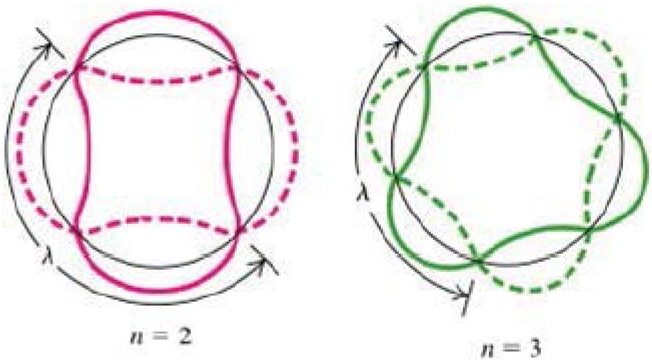
$$\boxed{L = mvr = \frac{nh}{2\pi}}$$

  - m is electron mass
  - v is orbital velocity
  - r is orbital radius
  - n is the principle quantum number (basically what orbital you're in – 1, 2, 3 etc)
  - h is Planck's constant
- ➔ The model was that the electron orbited the proton.
  - Centripetal force was by the Coulomb force.
  - Mass of the proton by far exceeded the electron and hence, electron orbital was roughly circular.

- Bohr's model could be related to de Broglie's hypothesis.

$$\boxed{\begin{aligned} mvr &= \frac{nh}{2\pi} \\ 2\pi r &= \frac{nh}{mv} \\ 2\pi r &= n\lambda \end{aligned}}$$

- ➔ This suggests that a certain number of wavelengths fit along the circumference.
- ➔ This implies that electrons are standing waves.



Newton's second law:

$$F_c = F_{EM}$$
$$\frac{e^2}{4\pi\epsilon_0 r_n^2} = \frac{m v_n^2}{r_n} \qquad (12)$$

- r_n and v_n are e^- orbital radius, speed

Quantisation:

$$m v_n r_n = \frac{nh}{2\pi}$$

- Eq. (13) $\Rightarrow$

$$v_n = \frac{nh}{2\pi m}\frac{1}{r_n}$$

Substituting Eq. (14) into (12):

$$\frac{e^2}{4\pi\epsilon_0 r_n^2} = \frac{m}{r_n}\frac{n^2 h^2}{4\pi^2 m^2 r_n^2}$$
$$\Rightarrow r_n = \frac{h^2\epsilon_0}{m\pi e^2} n^2$$

Substituting Eq. (16) into (14):

$$v_n = \frac{nh}{2\pi m}\frac{m\pi e^2}{h^2\epsilon_0 n^2}$$
$$\text{or} \quad v_n = \frac{e^2}{2h\epsilon_0}\frac{1}{n}$$

The radii are multiples of the smallest radius:

$$r_n = a_0 n^2 \quad \text{with} \quad a_0 = \frac{h^2\epsilon_0}{m\pi e^2} = r_1$$

- evaluating the Bohr radius a_0:

$$a_0 = \frac{h^2\epsilon_0}{m\pi e^2}$$
$$\approx \frac{\left(6.626\times10^{-34}\,\mathrm{J\,s}\right)^2 \times 8.854\times10^{-12}\,\mathrm{C\,N^{-1}\,m^{-1}}}{\pi\times 9.109\times10^{-31}\,\mathrm{kg}\times(1.602\times10^{-19}\,\mathrm{C})^2}$$
$$\approx 5.29\times10^{-11}\,\mathrm{m}$$

- which is consistent with atomic dimensions

Kinetic energy:

$$K_n = \tfrac{1}{2} m v_n^2 = \frac{me^4}{8\epsilon_0^2 h^2}\frac{1}{n^2} \qquad (20)$$

Potential energy:

$$U_n = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n} = -\frac{me^4}{4\epsilon_0^2 h^2}\frac{1}{n^2} \qquad (21)$$

So the total energy of the e^- is

$$E_n = K_n + U_n = -\frac{me^4}{8\epsilon_0^2 h^2}\frac{1}{n^2} \qquad (22)$$

- which we can rewrite as:

$$E_n = -\frac{hcR'}{n^2} \quad \text{where} \quad R' = \frac{me^4}{8\epsilon_0^2 h^3 c} \qquad (23)$$

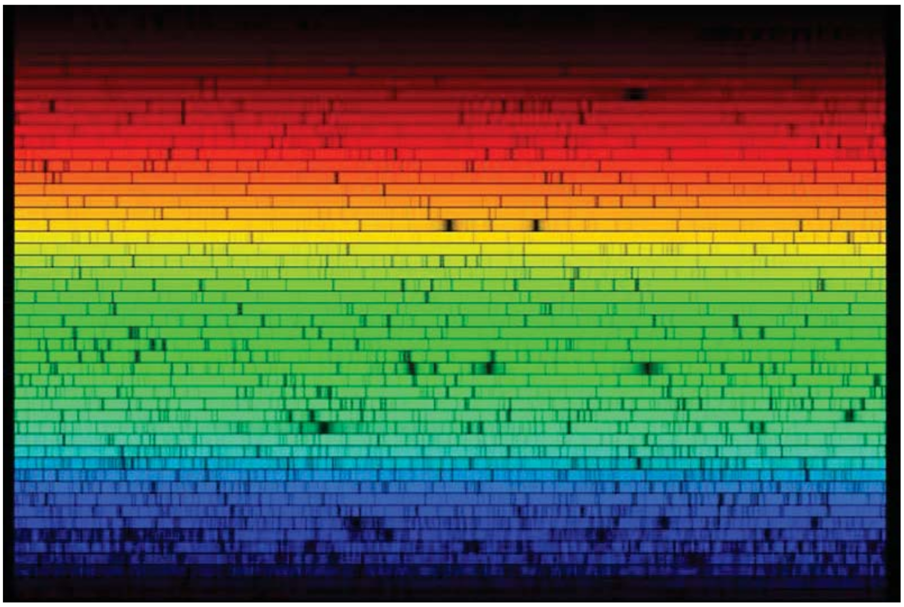
- and evaluating R':

$$\frac{me^4}{8\epsilon_0^2 h^3 c} \approx \frac{9.109\times10^{-31}\,\mathrm{kg}\times\left(1.602\times10^{-19}\,\mathrm{C}\right)^4}{8\times\left(8.854\times10^{-12}\,\mathrm{C\,N^{-1}\,m^{-1}}\right)^2\times\left(6.626\times10^{-34}\,\mathrm{J\,s}\right)^3\times 3\times10^8\,\mathrm{m/s}}$$

i.e. $R' \approx 1.097\times10^7\,\mathrm{m^{-1}}$

- so $R' \approx R$ [the Rydberg constant in the Balmer formula, Eq. (6)]

- Even though the Bohr model isn't correct, it correctly predicts the hydrogen spectrum.
  - ➔ Principal quantum number defines the energy state (n=1,2,3etc).
  - ➔ Protons aren't 100% stationary – they do move a little bit but not much compared to the electrons.
    - ▪ It's like how we say the Sun is stationary but in reality, it is still orbiting a centre of mass.
- The ground state is when the principle quantum number is n=1.
  - ➔ Decay back to the ground state results in photon emission.
  - ➔ Photon is emitted in a random direction.
  - ➔ If a continuous spectrum is shone through the gas, light is absorbed at specific wavelengths.
  - ➔ Absorbed light is re-emitted in random directions.
- The Sun's spectrum has many absorption lines but they are fairly difficult to see unless the spectrum is zoomed in.
  - ➔ Some light is absorbed by atoms in the Sun's atmosphere.
  - ➔ Absorption lines can be used to determine what elements are in the Sun's atmosphere.



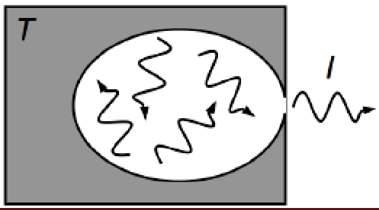
  - ➔ Helium was discovered based on the line spectrum of the Sun.
- Bohr's model also described single-electron atoms.
  - ➔ Ionised helium, doubly-ionised lithium etc.

$$\boxed{E_n \approx -\frac{Z^2(13.6\ eV)}{n^2}}$$

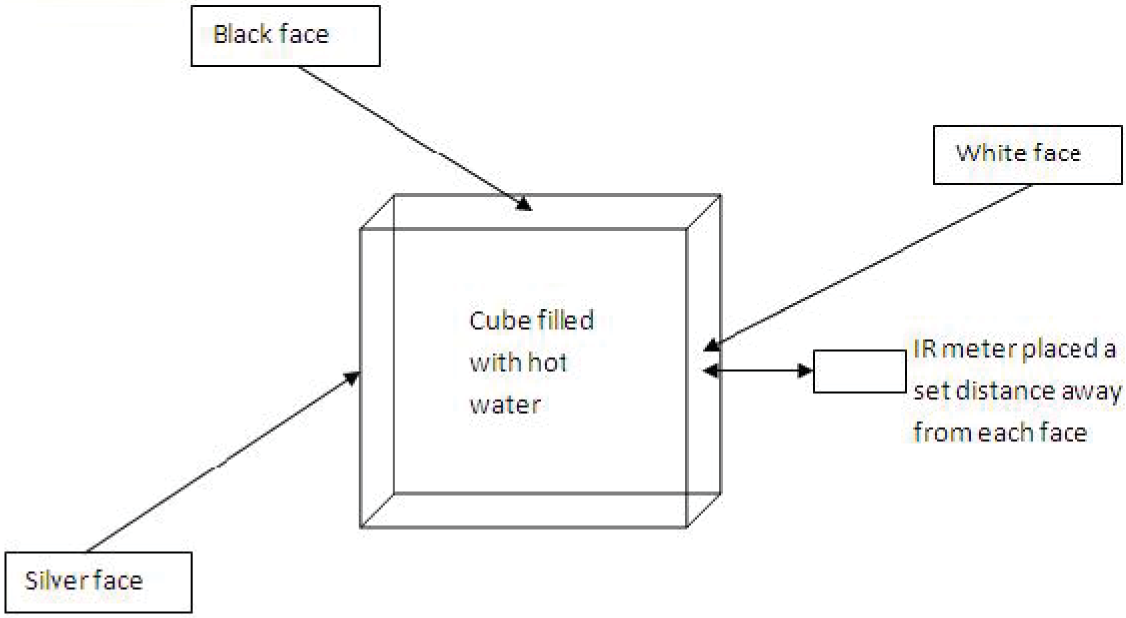
- Even though Bohr's model was useful, it is not good enough.
  - ➔ Orbits are unstable.
  - ➔ Electron orbits would form magnetic fields – not seen in hydrogen.

Lecture 8: Continuous Spectra

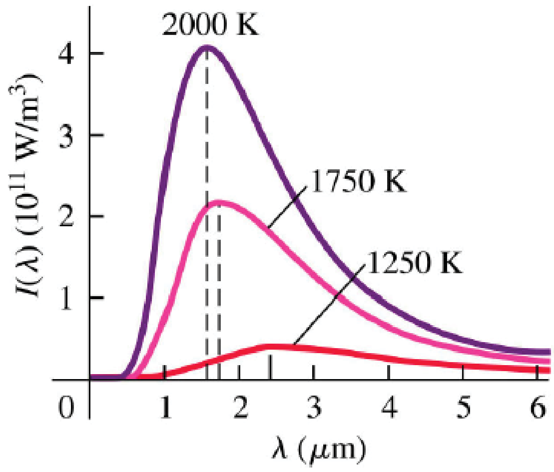
- Hot solids emit a continuous spectrum.
- A blackbody is one that absorbs almost all incoming radiation – it is also the best emitter of radiation.
  - ➔ Consider a cavity, if light falls into the cavity, it will probably be stuck inside there.



- A Leslie cube (below) can be used to show that the strongest absorber and emitter is the black face and the weakest absorber and emitter is the silver face.



  - ➔ Blackbodies are ideal emitters.
- $I_\lambda(T)$ is the power per unit area, per unit λ.
  - ➔ Has a single peak as a function of λ.
  - ➔ The peak shifts to shorter λ for spectra with higher T.
  - ➔ For higher T, spectrum is more intense for all λ.



- The wavelength of the peak of the blackbody curve is given by Wien's displacement law.

$$\boxed{\lambda_{max}T = 2.9 \times 10^{-3}}$$

- Stefan-Boltzmann law for total intensity:

$$\boxed{I(T) = \int_1^\infty I_\lambda(T)d\lambda = \sigma T^4}$$

  - ➔ σ is the Stefan-Boltzmann constant=5.67×10^{-8} W m^{-2} K^{-4}

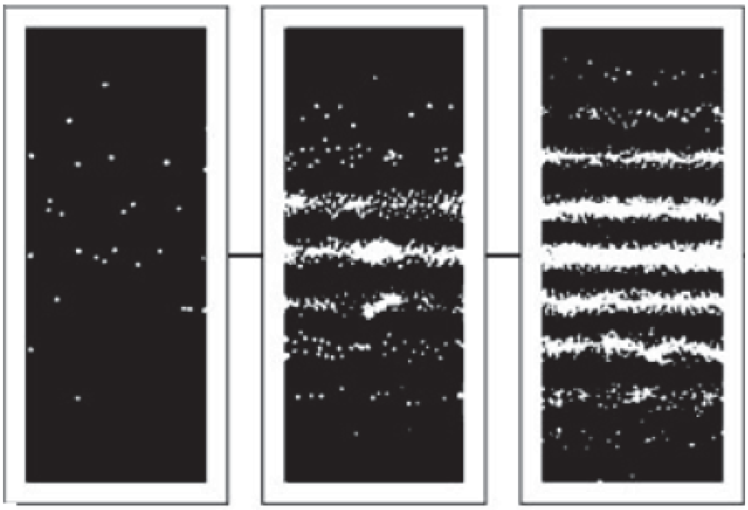
Lecture 9: Probability and Uncertainty

- In classical physics, electron is just a point and charged.
  - ➔ Its motion can be described as (x,v) or (x,p).
  - ➔ In quantum physics, there is uncertainty in the position and momentum of particles such as electrons.

$$\boxed{\begin{aligned} \Delta p_x \Delta x &\geq \frac{h}{4\pi} \\ \Delta p_y \Delta y &\geq \frac{h}{4\pi} \\ \Delta p_z \Delta z &\geq \frac{h}{4\pi} \\ \Delta E \Delta t &\geq \frac{h}{4\pi} \end{aligned}}$$

- ➔ Δt is the duration in energy state E.

- The diffraction pattern describes a statistical result – demonstrates the wave nature of electrons.
  - ➔ Electrons arrive randomly at the detector.
  - ➔ Pattern intensity corresponds to the probability of arrival at a point.
  - ➔ Below, electrons pass through one at a time.



After 28 electrons — After 1000 electrons — After 10,000 electrons

- If electrons pass through one at a time, what interferes? The electron with itself?
  - ➔ Bohr's model states that electrons orbit in a plane, say z=0.
  - ➔ Therefore, p_z=0 and Δp_z is 0.
  - ➔ By the Uncertainty principle, Δz=∞
  - ➔ As we know that z=0, Δz=0 – this is a contradiction.

Lecture 10: Wave Functions and the Schrödinger Equation

- Electrons cannot be described as point particles.
  - ➔ They don't have defined positions and velocities (Uncertainty Principle).
- The description of the position and motion of an electron can be described by a wave function (ψ(r,t)).
  - ➔ It describes where the particle is likely to be.
  - ➔ The wave function is the wave amplitude.
  - ➔ The wave function is a complex number at dxdydz at time t.
  - ➔ $|\psi(r,t)|^2$ is the probability if finding the particle at r(x,y,z) – probability density function (probability per volume)

$$\boxed{\int\int\int |\psi(\mathrm{r},\mathrm{t})| dxdydz = 1}$$

    - The probability of the particle being somewhere is 1.
    - This is called normalisation condition.

- Usually, the probability of finding a particle in a certain place changes with time.
  - ➔ But not always, such as electrons in defined energy levels – this makes $\Delta t \rightarrow \infty$
  - ➔ These situations are called stationary states.
- For a stationary state, the wave function has the form:

$$\boxed{\psi(\mathrm{r,t}) = \psi(\mathrm{r})e^{-iEt/\hbar}}$$

  - ➔ E is the energy.
  - ➔ Ψ(r) is the stationary-state wave function.
    - Using the above equation:

$$\boxed{|\psi(\mathrm{r,t})|^2 = |\psi(\mathrm{r})|^2}$$

  - ➔ Therefore, probability a particle is at a point is time-independent.
- Stationary states oscillate harmonically in time.

$$\psi(\mathrm{r,t}) = \psi(\mathrm{r})e^{-iEt/\hbar}$$
$$\psi(\mathrm{r,t}) = \psi(\mathrm{r})(\cos\frac{Et}{\hbar} - i\sin\frac{Et}{\hbar})$$
$$\psi(\mathrm{r,t}) = \psi(\mathrm{r})(\cos\frac{2Et\pi}{h} - i\sin\frac{2Et\pi}{h})$$
$$\psi(\mathrm{r,t}) = \psi(\mathrm{r})(\cos\frac{2hft\pi}{h} - i\sin\frac{2hft\pi}{h})$$
$$\psi(\mathrm{r,t}) = \psi(\mathrm{r})(\cos 2\pi ft - i\sin 2\pi ft)$$
$$\psi(\mathrm{r,t}) = \psi(\mathrm{r})(\cos\omega t - i\sin\omega t)$$

  - ➔ Ψ(r) has a fixed spatial form – they are standing waves.
- The 1D time-independent Schrödinger equation is:

$$\boxed{\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi = E\psi}$$

  - ➔ $\psi$is the 1D stationary-state wave function.
  - ➔ $m$is mass of the particle.
  - ➔ $E$is the total energy or the particle.
  - ➔ $U(x)$is the potential energy of the particle at position x.
- When there are no forces on a particle, the potential energy is zero – U(x)=0.

$$\therefore \frac{d^2\psi}{dx^2} = \frac{-2m}{\hbar^2}E\psi$$
Solution is in the form: $\psi = Ae^{ikx}$
$$\frac{d^2\psi}{dx^2} = -k^2x$$
By equating: $k = \pm\frac{(2mE)^{\frac{1}{2}}}{\hbar}$

- For a free particle, the total energy is:

$$E = K = \frac{1}{2}mv^2$$
$$p = mv$$
$$\therefore E = \frac{p^2}{2m}$$

- Phase velocity of a wave is the rate at which the phase of the wave propagates in space.

$$v_\emptyset = \frac{\omega}{k}$$
$$\psi(\mathrm{r,t}) = Ae^{-i(kx+\omega t)}$$