

## QM2 Revision

Test	P/ NP	Assumptions/Conditions	Confidence interval	For 1-tail and 2-tail tests?	Hypotheses	Test statistic (under H <sub>0</sub> )	df	Decision rule	p-value reported by Eviews
Matched pairs	P	<ol style="list-style-type: none"> <li>Random sample</li> <li>Quantitative variable measured on interval or ratio scale</li> <li>If n is large (≥30), CLT holds, i.e. the distribution of the sample mean difference is approximately normally distributed when <math>\sigma</math> is known. However, if <math>\sigma</math> is unknown, we should use the t distribution. Therefore, we assume</li> </ol>	$\bar{X}_D \pm z_{\alpha/2} \sigma_{\bar{x}_D}$ $\bar{X}_D \pm t_{\alpha/2} s_{\bar{x}_D}$	Y	H <sub>0</sub> : $\mu_D=0$ H <sub>A</sub> : $\mu_D \neq 0$ $\mu_D > 0$ or $< 0$	$Z = \frac{\bar{X}_D - \mu_{D,0}}{\sigma_{\bar{x}_D}}$ $t = \frac{\bar{X}_D - \mu_{D,0}}{s_{\bar{x}_D}}$	n-1	<p><b>If <math>\sigma_D</math> is known:</b></p> RT: Reject H <sub>0</sub> if - $Z_{obs} > Z_{\alpha}$ - p-value < $\alpha$	2-sided Prob./2 for 1-tail
								LT: Reject H <sub>0</sub> if - $Z_{obs} < -Z_{\alpha}$ - p-value < $\alpha$	
								2T: Reject H <sub>0</sub> if - $ Z_{obs}  > Z_{\alpha/2}$ - p-value < $\alpha$	
								<p><b>If <math>\sigma_D</math> is unknown:</b></p> RT: Reject H <sub>0</sub> if - $t_{obs} > t_{\alpha,df}$ - p-value < $\alpha$	
								LT: Reject H <sub>0</sub> if - $t_{obs} < -t_{\alpha,df}$ - p-value < $\alpha$	
								2T: Reject H <sub>0</sub> if - $ t_{obs}  > t_{\alpha/2,df}$ - p-value < $\alpha$	

		that the sampled population {D <sub>i</sub> } is not extremely non-normal							
Sign test (based on the signs of the observed deviations from the hypothesized median)	NP	1. data is a random sample 2. measurement scale is at least ordinal	N/A	Y	H <sub>0</sub> : $\eta_1 - \eta_2 = 0$ H <sub>A</sub> : $\eta_1 - \eta_2 \neq 0$ or $> 0$ or $< 0$	<ul style="list-style-type: none"> <li>- RT: S+</li> <li>- LT: S-</li> <li>- 2T: S- or S+, where S- and S+ are the numbers of +ve and -ve deviations</li> </ul> <p>If H<sub>0</sub> is true, S follows a <b>binomial</b> distribution. =&gt; <b>S ~ B(n, 0.5)</b>, where <b>n is number of non-zero deviations</b></p> <p>Eviews uses the max( S-, S+) as the test statistic.</p> <p>For <b>n ≥ 10</b>, the binomial distribution</p>	N/A	<p><b>When n is small</b></p> <p>RT: Reject H<sub>0</sub> if</p> <ul style="list-style-type: none"> <li>- S<sub>+</sub> &gt; upper CV</li> </ul> <p>LT: Reject H<sub>0</sub> if</p> <ul style="list-style-type: none"> <li>- S<sub>-</sub> &lt; lower CV</li> </ul> <p>2T: Reject H<sub>0</sub> if</p> <ul style="list-style-type: none"> <li>- S is below lower CV or above upper CV</li> </ul> <p><b>When n is large enough,</b></p> <p>RT: Reject H<sub>0</sub> if</p> <ul style="list-style-type: none"> <li>- Z<sub>obs</sub> &gt; Z<sub>α</sub></li> <li>- p-value &lt; α</li> </ul> <p>LT: Reject H<sub>0</sub> if</p> <ul style="list-style-type: none"> <li>- Z<sub>obs</sub> &lt; - Z<sub>α</sub></li> <li>- p-value &lt; α</li> </ul> <p>2T: Reject H<sub>0</sub> if</p> <ul style="list-style-type: none"> <li>-  Z<sub>obs</sub>  &gt; Z<sub>α/2</sub></li> <li>- p-value &lt; α</li> </ul>	2-sided Prob./2 for 1-tail

can be approximated with a normal distribution (use continuity correction) Eviews always performs the sign test with normal approximation, which is reasonable only for relatively large sample sizes. Thus, it can be misleading when sample size is small.

Consequently, check the exact binomial sign test instead.

$$B(n, p) \sim N(np, \sqrt{npq})$$

$$S \sim N(0.5n, 0.5\sqrt{n})$$

$$Z = \frac{S - 0.5n}{0.5\sqrt{n}}$$

<p><b>Wilcoxon signed ranks test</b>          Considers not only the signs of the deviations from the median but their magnitudes as well. Based on the ranks of the absolute values of the non-zero deviations</p>	<p>NP</p>	<ol style="list-style-type: none"> <li>1. Data is a random sample</li> <li>2. Variable of interest is quantitative and continuous</li> <li>3. Measurement scale is interval or ratio</li> <li>4. Distribution of sampled population is symmetric</li> </ol>	<p>N/A</p>	<p>Y</p>	<p><math>H_0: \eta_1 - \eta_2 = 0</math>  <math>H_A: \eta_1 - \eta_2 \neq 0</math> or <math>&gt; 0</math> or <math>&lt; 0</math></p>	<p><math>T_- + T_+ = \frac{n(1+n)}{2}</math>,          where <math>T_-</math> and <math>T_+</math> denote the sums of ranks that belong to -ve and +ve deviations</p> <p>When <math>H_0</math> is correct,  <math>T_- = T_+ = \frac{n(1+n)}{4}</math>          where <b>n is number of non-zero deviations</b></p> <p>2T: <math>T_-</math> or <math>T_+</math>          LT: <math>T_-</math>          RT: <math>T_+</math></p> <p><math>T</math> has a non-standard sampling distribution. For <math>6 \leq n \leq 30</math>, critical values are found in WSRT Table.</p> <p>If <b><math>n \geq 30</math></b>, sampling distribution of <math>T</math> can be approximated</p>	<p>N/A</p>	<p><b>When n is small</b>          RT: Reject <math>H_0</math> if          - <math>T_+ &gt; \text{upper CV}</math></p> <p>LT: Reject <math>H_0</math> if          - <math>T_- &lt; \text{lower CV}</math></p> <p>2T: Reject <math>H_0</math> if          - <math>T</math> is below lower CV or above upper CV</p> <p><b>When n is large enough,</b>          RT: Reject <math>H_0</math> if          - <math>Z_{\text{obs}} &gt; Z_\alpha</math>          - <math>p\text{-value} &lt; \alpha</math></p> <p>LT: Reject <math>H_0</math> if          - <math>Z_{\text{obs}} &lt; -Z_\alpha</math>          - <math>p\text{-value} &lt; \alpha</math></p> <p>2T: Reject <math>H_0</math> if          - <math> Z_{\text{obs}}  &gt; Z_{\alpha/2}</math>          - <math>p\text{-value} &lt; \alpha</math></p>	<p>2-sided          Prob./2 for 1-tail</p>
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with a normal distribution.

$$T \sim N(\mu_T, \sigma_T)$$

$$Z = \frac{T - \mu_T}{\sigma_T}$$

$$\mu_T = \frac{n(n+1)}{4}$$

$$\sigma_T^2 = \frac{n(n+1)(2n+1)}{24}$$

Eviews always performs the WSRT with normal approximation, which is reasonable only when the number of non-zero deviations from hypothesised median is at least 30 ( $n \geq 30$ ). Thus, it can be misleading when sample size is small.

<p>2-independent-sample Z/t test</p>	<p>P</p>	<ol style="list-style-type: none"> <li>1. Samples are random and independent</li> <li>2. Sampled populations are normally distributed</li> <li>3. Population variances are unknown and equal/different</li> </ol>	<p><b>CASE 1:</b>  <math>(\bar{X}_1 - \bar{X}_2) \pm z_{\alpha/2} \sigma_{\bar{X}_1 - \bar{X}_2}</math></p> <p><b>CASE 2 &amp; 3:</b>  <math>(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} s_{\bar{X}_1 - \bar{X}_2}</math></p>	<p>Y</p>	<p>H<sub>0</sub>: μ<sub>1</sub>-μ<sub>2</sub>=0  H<sub>A</sub>:  μ<sub>1</sub>-μ<sub>2</sub>≠0 or  &gt; 0 or &lt; 0</p>	<p><b>CASE 1:</b>  <math>Z = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_{D,0}}{\sigma_{\bar{X}_1 - \bar{X}_2}}</math></p> $\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ <p><b>CASE 2:</b>  <math>t = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_{\bar{X}_1 - \bar{X}_2,0}}{s_{\bar{X}_1 - \bar{X}_2}}</math></p> $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ $s_{\bar{X}_1 - \bar{X}_2} = \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$ $df = n_1 + n_2 - 2$ <p><b>CASE 3:</b>  <math>t = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_{\bar{X}_1 - \bar{X}_2,0}}{s_{\bar{X}_1 - \bar{X}_2}}</math></p> $s_{\bar{X}_1}^2 = \frac{s_1^2}{n_1}, \quad s_{\bar{X}_2}^2 = \frac{s_2^2}{n_2}$ $s_{\bar{X}_1 - \bar{X}_2} = \sqrt{s_{\bar{X}_1}^2 + s_{\bar{X}_2}^2}$ $df = \frac{(s_{\bar{X}_1 - \bar{X}_2}^2)^2}{(s_1^2)^2 / (n_1 - 1) + (s_2^2)^2 / (n_2 - 1)}$	<p>Depends on which case</p>	<p><b>CASE 1:</b>  RT: Reject H<sub>0</sub> if  - Z<sub>obs</sub> &gt; Z<sub>α</sub>  - p-value &lt; α</p> <p>LT: Reject H<sub>0</sub> if  - Z<sub>obs</sub> &lt; - Z<sub>α</sub>  - p-value &lt; α</p> <p>2T: Reject H<sub>0</sub> if  -  Z<sub>obs</sub>  &gt; Z<sub>α/2</sub>  - p-value &lt; α</p> <p><b>CASE 2 &amp; 3:</b>  RT: Reject H<sub>0</sub> if  - t<sub>obs</sub> &gt; t<sub>α,df</sub>  - p-value &lt; α</p> <p>LT: Reject H<sub>0</sub> if  - t<sub>obs</sub> &lt; - t<sub>α,df</sub>  - p-value &lt; α</p> <p>2T: Reject H<sub>0</sub> if  -  t<sub>obs</sub>  &gt; t<sub>α/2,df</sub>  - p-value &lt; α</p>	<p>2-sided  Prob./2 for 1-tail</p>
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Eviews performs 4 parametric tests. First t-test assumes populations

						variances are equal. Satterthwaite-Welch t-test allows for unequal population variances.			
Wilcoxon rank-sum test	NP	<ol style="list-style-type: none"> <li>Data consists of 2 independent random samples drawn from</li> <li>2 populations that differ at most wrt to their central locations, i.e. they should be identical in shape and spread</li> <li>Measurement scale is at least ordinal</li> <li>Variable of interest is continuous</li> </ol>	N/A	Y	$H_0: \eta_1 = \eta_2$ $H_A:$ $\eta_1 \neq \eta_2$ $\eta_1 > \eta_2$ $\eta_1 < \eta_2$	<p>Either <math>T_1</math> or <math>T_2</math>  If <math>n_1 = n_2</math>, <math>T_1</math> and <math>T_2</math> leads to the same absolute value.</p> <p>Sampling dist is non-standard  Exact small samples CVs when <math>n_1 \leq 10</math>, <math>n_2 \leq 10</math> are found in the WRS Table.</p> <p>When <math>H_0</math> is true and when <b><math>n_1 &gt; 10</math>, <math>n_2 &gt; 10</math></b>, sampling dist of <math>T_i</math> (<math>i=1</math> or <math>2</math>) can be approximated with a normal distribution.</p> $T \sim N(\mu_T, \sigma_T)$	N/A	<p><b>When n is small</b>  RT: Reject <math>H_0</math> if  - <math>T_1 &gt; \text{upper CV}</math></p> <p>LT: Reject <math>H_0</math> if  - <math>T_1 &lt; \text{lower CV}</math></p> <p>2T: Reject <math>H_0</math> if  - <math>T</math> is below lower CV or above upper CV</p> <p><b>When n is large enough,</b>  RT: Reject <math>H_0</math> if  - <math>Z_{\text{obs}} &gt; Z_\alpha</math>  - <math>p\text{-value} &lt; \alpha</math></p> <p>LT: Reject <math>H_0</math> if  - <math>Z_{\text{obs}} &lt; -Z_\alpha</math>  - <math>p\text{-value} &lt; \alpha</math></p> <p>2T: Reject <math>H_0</math> if  - <math> Z_{\text{obs}}  &gt; Z_{\alpha/2}</math>  - <math>p\text{-value} &lt; \alpha</math></p>	2-sided Prob./2 for 1-tail

$$\mu_T = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma_T^2 = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$

$$Z = \frac{T - \mu_T}{\sigma_T}$$

If sample sizes are small, Eviews would still approximate with a normal distribution, and can be misleading.