FNCE30007: DERIVATIVE SECURITIES

MID-SEMESTER AND FINAL EXAM NOTES

INTRODUCTION TO FUTURES

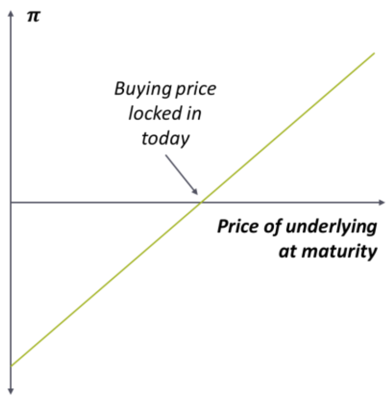
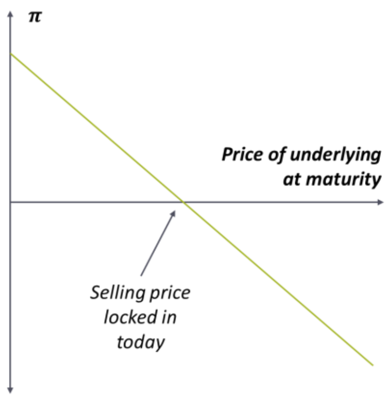
Futures: a contract between two parties (buyer and seller), where one party buys something from the other at a later date, for a price agreed today

- Exchange traded → subject to daily settlement of gains and losses → guaranteed against default risk
- Available on a range of underlying securities e.g. bonds, shares, indices such as SFE SPI 200

$t = 0$	$t = T$
F_0 ← futures or forward price that we agree to buy and sell at, but at time T S_0 ← spot price of the underlying today F_0 and S_0 are different prices linked through arbitrage	Maturity date Buying and selling occurs

PROFIT FROM LONG AND SHORT FORWARD OR FUTURES POSITION

- Symmetric payoffs (unlike with options)
- Trade at settle price, less π in margin account

	LONG POSITION (BULLISH)	SHORT POSITION (BEARISH)
Payoff diagram		
If sold before maturity	$\boldsymbol{Payoff = \#\ of\ contracts\ (S - F_t)}$	$\boldsymbol{Payoff = \#\ of\ contracts\ (F_t - S)}$

SPECIFICATIONS

- **What can be delivered** (the asset) – most contracts are cash settled at expiry, but some are deliverable (90-day BABs and some commodities – usually only hedgers take delivery)
    - If not specified properly, then short would deliver cheapest asset, and once this was well-known no one would be prepared to go long
- **Where** it can be delivered; **When** it can be delivered (delivery months); **Contract size; Price quotes, price limits and position limits**

Most activity is in the nearest contract because of spreads – for far away contracts there is such a low demand that the spread is high → those with lower term to maturity have lower spreads (rolling contract)

- Spreads are narrow when margins are high → lower transaction costs
- When plotting prices over long time – splice prices together from 2 weeks from end of contract

Rolling (long) contract

OPENING AND CLOSING A CONTRACT

Open a position through broker or online trading account → close it by entering into opposite trade

If contract is not closed out prior to expiration:

- If **cash settled** → exchange closes out the position, and you are left with margin account balance
- If **deliverable** → settled by delivering assets underlying contract at settle price at maturity
- When there are alternatives about what is delivered, where, when, party with short position chooses → more options means lower futures price

Why not just speculate on the index through an ETF? Extremely high leverage through contracts → higher impact

CONVERGENCE OF FUTURES PRICE TO SPOT PRICE

As futures approach expiration →futures price converges to spot price, otherwise there's an arbitrage opportunity

$F_T = S_T$ at maturity, otherwise (e.g. if $F_T > S_T$) you would just buy at S_T, sell at F_T and pocket the difference (subject to transaction costs)

- Can arbitrage at any time – this is why there is only a little gap between spot and futures

MARGINS

- When two investors enter a trade they are exposed to **default risk** → role of exchange is to organise trading so that this risk is minimised
- A ***margin*** is cash or marketable securities deposited by an investor with his or her broker → balance in the margin account is adjusted to reflect daily settlement (marking to market)
- Margins minimise the possibility of a loss through a default on a contract
- If drop below maintenance margin, get a margin call → must **top up to original margin requirement**

STEP 1	$Margin\ account\ balance\ (day\ 1)\ = Initial\ margin + daily\ gain\ (loss)$
STEP 2	$Daily\ gain\ (loss) = number\ of\ contracts \times contract\ size \times \boldsymbol{price\ change}\ per\ contract\ per\ unit$

PRICE AND TRADING INFORMATION

Open: price when futures contract starts trading
High: highest price during the day
Low: lowest price during the day
Last: the last traded price during the day
Sett: the daily settlement price declared by the exchange at which all contracts are markets to market. Usually midpoint of closing bid and offer – may be different from last traded price

Settlement change: different between yesterday's and today's settlement price
Open interest – number of long (short) positions/contracts open – haven't been closed out
Volume: number of purchases (sales) during a specified period

When a new trade occurs what are the possible effects on open interest? If both sides of the transaction are entering a new contract open interest increases by 1. If both sides are closing out, open interest decreases by 1. If one party is entering a new contract and the other is closing out, open interest is unchanged.

Can the volume of trading in a day be greater than the open interest? Yes. 1) Where market is dominated by intraday trading 2) Everyone is trying to close out (near $t = T$) → taking the opposite position through trading → high volume

REGULATION

*Designed to protect public – e.g. through **price limits** – also **a partial substitute for margins***

- 0.45 price limit on \$5 stock means it can only trade in range of $(\$4.55,\ \$5.45)$ - If bad news overnight, and market hits lower bound, then: (1) in some markets it just can't go below 4.55 all day, (2) trading halt for an hour (3) trading halt all day
- Australian Regulators: ASIC, ACCC

How are price limits a partial substitute for margins? Margins are where the exchange wants to ensure that all parties have enough \$ in margin account to meet worst possible loss for 1 day. Price limits → reduce the size of the maximum loss → exchange can reduce margin requirement → meaning greater liquidity (of futures contract) – don't need to post as much to take a position → **therefore don't need as much in margin account**

FORWARDS V. FUTURES

***Forward:** an OTC agreement between two parties for one party to buy something from the other at a later date at a price agreed upon today → **no daily settlement** – at the end of the life of the contract one part buys the asset for the agreed price from the other party*

Forward	Futures
Private contract between two parties	Traded on an exchange
Not standardized	Standardised
Usually one specified delivery date	Range of delivery dates
Settled at end of contract	Settled daily
Delivery or final settlement usual	Usually closed out prior to maturity
Some credit risk	Virtually no credit risk
No upfront costs	Require upfront payment into margins account

HEDGING WITH FUTURES

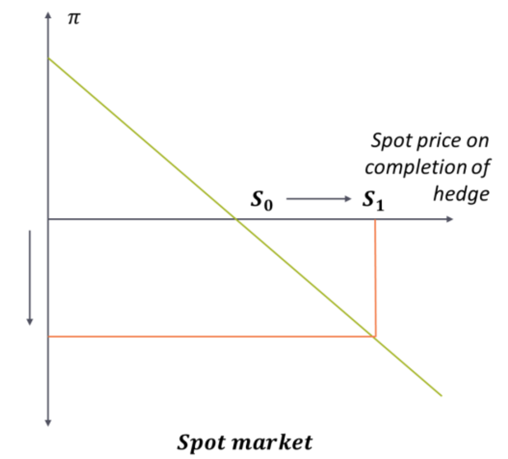
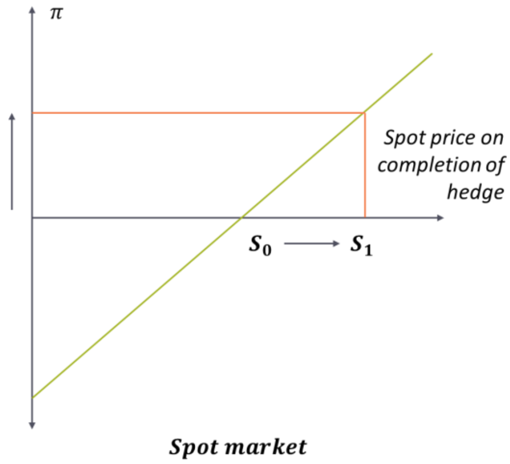
Previous week focussed on speculators, this week focusses on hedgers

SHORT AND LONG HEDGES

Objective: take a futures position that minimises risk as far as possible

- Always assume that the actual transaction is occurring in the spot market → to combat the risk of this actual transaction occurring at an unknown spot price we take out a futures contract that is opposite
- Hedgers will take out whatever their aim is in the futures market (i.e. buy or sell)

	LONG POSITION (PURCHASING ASSET)	SHORT POSITION (SELLING ASSET)

<table>
<tr><td>Underlying (spot) position</td><td>
</td><td>
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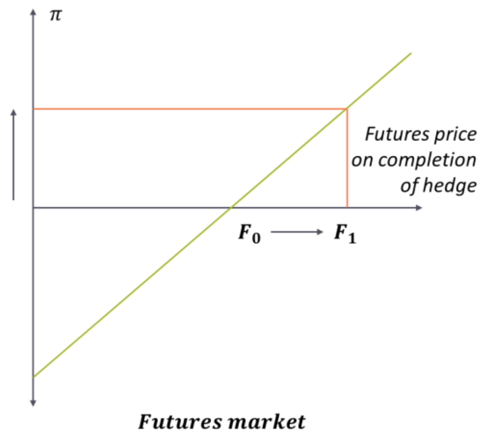
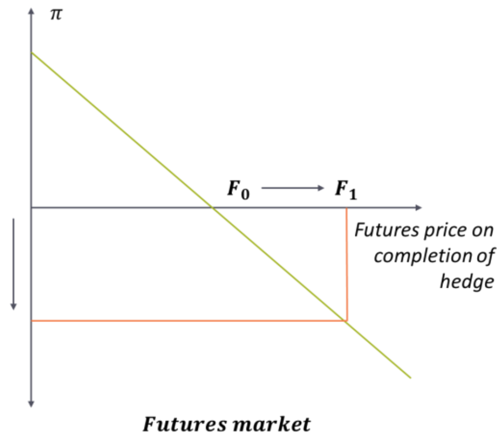
<tr><td>Futures position</td><td>
</td><td>
</td></tr>
<tr><td>When they hedge</td><td>Purchasing an asset in the future and want to lock in the price</td><td>Selling an underlying asset in the future and want to lock in the price
Asset is owned or will be owned</td></tr>
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3 APPROACHES TO PROVING PRICE GUARANTEE

*It **is only because $S_T = F_T$** that a hedger will be guaranteed the price → prove this with 3 approaches, each giving a different insight*

Short hedge = $(S_T - S_0) + (F_0 - F_T)$ → If we have until maturity we know that $F_T = S_T$ so we know we can lock in our costs at $F_0 - S_0$

- This is a perfect hedge but this is not always the case, may not be identical asset, and date being traded and expiry may differ

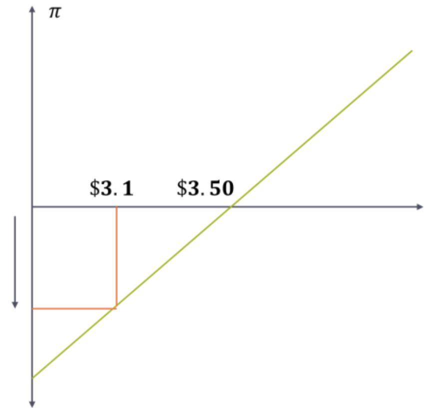
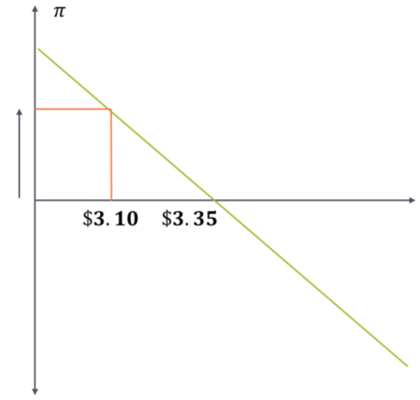
EXAMPLE:

- April 20: Farmer negotiates to sell 50,000 bu of corn at the spot price on June 20
- June 20 is futures maturity date – no basis risk
- Quotes: **Spot price of corn: $3.50/bu**; **June corn futures price: $3.35/bu** (each contract is for 5,000 bu)
- After futures gains & losses the price received by the farmer should be $3.35/bu.

If price decreases to $3.10 – also calculate if price increases to $3.70 – not shown below but same method

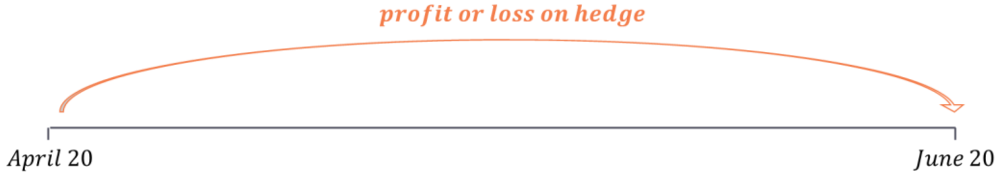
APPROACH 1: VALUING SPOT AT MATURITY (WHAT ACTUALLY HAPPENS)

Spot	*Futures*	*Calculation method*

		***Think about inflows and outflows*** ***Values spot at t = T*** $S_2 + (F_1 - F_2)$
Sell in spot for $3.10 50,000 × 3.1 =*$150,000*	*Gain $0.25 in futures* 50,000 × 0.25 = $12,500	$\mathbf{150,000 + 12,500}$ $= \$\mathbf{167,500}$

Locked in price of $3.35 (167,500/50,000) with **absolute certainty**

APPROACH 2: VALUING SPOT AT T = 0



Position @ Apr. 20 → 50,000*bu.* @$3.50 → **$175,000**

Adjust for profit or loss over 2 month window

profit or loss on hedge:

spot: 50,000 × (3.1 − 3.5) → **(20,000)**

futures: 50,000 × (3.35 − 3.1) → **12,500**

Profit or loss on hedge = **(7,500)**

Add value of spot at t = 0 → **$175,000**

= **$167,500**

$$\textbf{\textit{In both approaches, price per bushel}} = \frac{\$167{,}500}{50{,}000} = \$\mathbf{3.35}$$

Question: Should the food company in the example buy the corn it needs today in the spot market?	
$\boldsymbol{If\ F(t) < S(t)}$	If the company doesn't need corn until June 20, better not to buy today → a lower price will be paid on June 20 ($3.35 vs. $3.50), and storage costs are avoided
$\boldsymbol{If\ F(t) > S(t)}$	Can't just compare prices, as there are other factors $Adjust\ spot\ price\ by: storage\ costs + \ time\ value\ of\ \$ - convenience\ yield$ Convenience yield: may be benefits of having something in stock – e.g. if spike in corn market, can sell off, or if there is a supplier issue

- Most of the time long hedgers do not take delivery, they close out before the delivery date and buy in the spot → partly because delivery arrangements can be very expensive

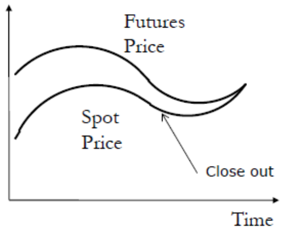
**See basis risk section for approach 3*

SHOULD COMPANIES HEDGE?

Arguments in favour of hedging:	**Arguments against hedging**
Companies **should focus on the main business** they are in and take steps to minimise risks arising from interest rates, exchange rates and other market variables	**Shareholders are usually well-diversified** and can make their own hedging decisions It may **increase risk to hedge** when competitors do not → when perfectly competitive → stable profit margins → pass gains or losses onto consumer → Therefore, there is no spot exposure, but if prices reduce you can be subject to margin calls etc.

• e.g. Shareholders in BHP don't want to be exposed to interest rates, just commodity prices	Explaining a situation where there is a loss on the hedge and a gain on the underlying can be difficult

BASIS RISK



Gap between futures and spot price at any given time → if you close out at the end, then it is perfectly predictable

- ***Perfect hedge*** completely eliminates risk (see previous examples)
- Most hedges are **imperfect** because
  - Hedge requires the futures contract to be closed out before expiration date
  - Hedger may not be sure about exact date the asset will be bought or sold
  - ***Cross hedge:*** asset to be hedged not the same as the asset underlying the futures contract
- *Above problems create* ***basis risk****:*

$$\text{Basis(t)} = spot\ price\ of\ asset\ to\ be\ hedged\ (t) - futures\ price\ of\ contract\ used\ (t)$$
$$Basis_2 = S_2 - F_2$$

- *Arises because of uncertainty about the basis when the hedge is closed out*
- *Want to replace spot risk with basis risk when hedging → want spot risk > basis risk*

	LONG HEDGE	SHORT HEDGE
F_1: ***initial futures price*** F_2: ***final futures price*** S_2: ***final asset price*** **Unknown at time t = 1*	$\boldsymbol{Cost\ of\ asset} = S_2 - (F_2 - F_1) = \boldsymbol{F_1} + \boldsymbol{Basis_2}$	$\boldsymbol{Price\ realised} = S_2 + (F_1 - F_2) = \boldsymbol{F_1} + \boldsymbol{Basis_2}$

APPROACH 3: ADJUST FUTURES PRICE FOR BASIS

$$If\ price\ fell\ to\ \$3.1 \rightarrow F_1 + Basis_2 = F_1 + (S_2 - F_2)$$
$$= \$3.35 + (3.1 - 3.1) = \$3.35$$

- No basis risk because perfectly hedged

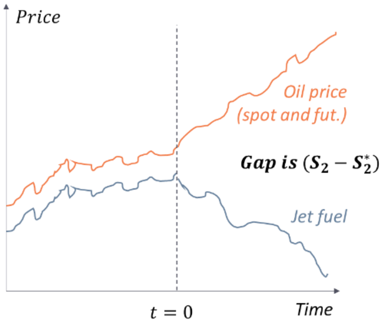
CROSS HEDGE

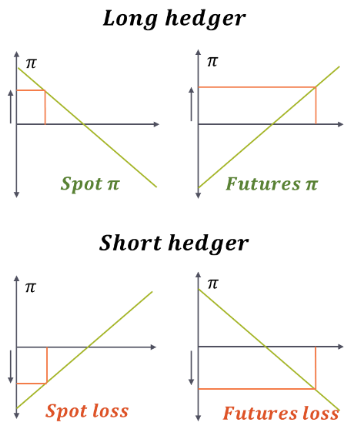
Hedging by taking a position in ***related*** *futures contract (done when no derivatives contract for asset being hedged or the futures contract exists but market is highly illiquid)*

- Success depends on the relationship between the asset being hedged and the asset which underlies the derivatives contract → should cointegrated with fast rate of equilibrium adjustment
  - If you're going to cross hedge, then the gap has to be stable → adds additional source of risk

$$F_1 + (S_2^* - F_2) + (S_2 - S_2^*)$$
$$(S_2^* - F_2) = basis\ that\ exists\ if\ the\ asset\ being\ hedged\ were\ the\ same\ as\ the\ asset\ underlying\ the\ futures$$
$$(S_2 - S_2^*) = basis\ that\ arises\ from\ the\ difference\ between\ the\ two\ assets$$
$$\rightarrow additional\ source\ of\ uncertainty$$

WHAT HAPPENS IF GAP WIDENS?





If Qantas wanted to hedge jet fuel prices with oil futures:

$$F_1 + (S_2^* - F_2) + (S_2 - S_2^*)$$
$$F_1 + (S_2 - F_2)$$
$$F_1 + B_2$$

INDEX FUTURES HEDGING

WHEN IS THIS DONE? (1) *Used when you **want to be out of the market for a short period of time**, as hedging may be cheaper than selling the portfolio and buying it back; (2) Or used **when you want to hedge systematic risk** → if you feel that you have picked stocks that will outperform the market (without idiosyncratic risk)*

To hedge the risk in a portfolio **the number of contracts shorted is:**

$$\boldsymbol{Number\ of\ contracts\ shorted} = \beta \frac{P}{F}$$

$$P: portfolio\ value, \beta: portfolio\ beta,\ \ F: current\ value\ of\ one\ futures\ (futures\ price \times contract\ size)$$

- $\frac{P}{F}$ *is the number of contracts for index fund*
- If **beta = 1, portfolio return mirrors market return**; if **beta = 2, portfolio return tends to be twice** the market return, if **beta = 0.5, portfolio return tends to be half** the market return
- This is a special case of the 'Changing beta' formula (next), for if you want to close out all risk

CHANGING BETA

Used when you may not want to hedge all risk ($\downarrow \beta$)*, or if you want more weight into more aggressive stocks* ($\uparrow \beta$)

$$\boldsymbol{Number\ of\ contracts\ shorted} = (\beta - \beta^*)\frac{P}{F}, \qquad where\ \beta^* = target\ beta$$

MINIMUM VARIANCE HEDGE RATIO (MVHR)

Hedge ratio: *ratio of size (dollar value) of the position taken in futures contracts to the size (D.V.) of the exposure*

- So far we have set the hedge ratio at 1, but may be **sub-optimal to set hedge ratio to 1 when there is basis risk** (i.e. from cross hedging and/or when hedge completion date is not the same as futures expiration date)

$$\boldsymbol{Minimum\ Variance\ Hedge\ Ratio} = \boldsymbol{h^* = \rho \frac{\sigma_S}{\sigma_F}} = \frac{\sigma_{sf}}{\sigma_f^2}$$

$$where\ \sigma_S = standard\ deviation\ of\ \Delta S, \sigma_F = standard\ deviation\ of\ \Delta F, and\ \rho = correlation\ between\ \Delta S\ and\ \Delta F, \qquad \rho_{SF} = \frac{\sigma_{SF}}{\sigma_S \sigma_P}$$

- $h^* = \frac{your\ futures\ position\ in\ \$}{\$\ in\ spot} \rightarrow if\ h^* = 0.9, for\ every\ \$1\ in\ spot, you\ should\ short\ 90c\ (\$9m)$