FINC3017

INVESTMENTS AND PORTFOLIO MANAGEMENT

LECTURE 1: INTRODUCTION TO INVESTMENTS

WHAT IS AN ASSET?

- Equities
- Bonds
- Property
- Superannuation fund
- Wine
- Education
- Etc…

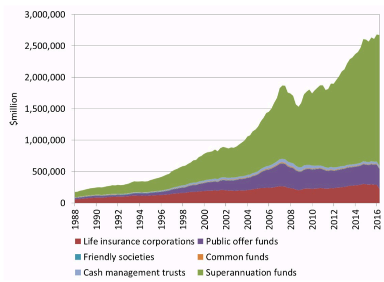
MARKET PARTICIPANTS

- **Participants**: individuals, investment managers (funds), insurance companies, superannuation / pension funds, banks, governments, sovereign wealth funds (e.g. Future Fund), universities (endowments), hedge funds, high frequency traders, market makers and dealers

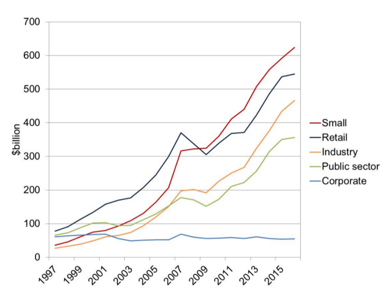
INVESTMENT FUND ASSET ALLOCATION

- Managed funds fall into a number of categories that pool investors' funds
- ***Unit trusts***:
    - Investors' funds are pooled, usually into specific types of assets
    - Investors are assigned units in the fund which are typically traded
    - Invest in tradable units → e.g. listed property trusts
        - Sell a fixed number of units to investors, closed end funds
    - An unlisted trust can issue new units at any time
        - Value of each unit depends on the value of the underlying investments
        - Open-ended funds → mutual funds in US
- ***Superannuation funds:*** accept and manage contributions from employers and / or employees to provide retirement income benefits
    - Aka pension funds in global market
    - Managed by life insurance companies, fund administrators, master trusts, pooled trusts (wholesale investment vehicle for other super funds)
    - Superannuation fund structure:
        - Defined benefit
            - Retirement payout is determined based on a formula
        - Defined contribution – aka accumulation funds
            - Value of retirement payout depends on investments of contributions in the fund
- ***Hedge funds***: seek to hedge against risky price movements via short selling, arbitrage trading, derivatives, distressed securities, low-grade bonds, and high leverage portfolios to maximise the expected return-risk of the portfolio
    - Access to hedge funds is limited
- ***Exchange traded funds (ETFs):***
    - Listed on the stock exchange
    - Trade as per any stock, unlike other managed funds
    - Essentially a hybrid between a listed security and open-ended fund
    - Provide ease of access and low costs of entry / exit
    - Often have an explicit objective and benchmark (e.g. index trading)
- TOP GRAPH:
    - Proportion of superannuation funds has grown immensely since 1988
    - Life insurance has seen some growth
    - Also some fairly decent growth in public offer funds
- BOTTOM GRAPH:
    - Corporate super funds haven't growth too much
    - All other classes have seen some significant growth
    - 'Small' refers to self-managed superannuation funds (SMSFs)

Assets under management ($m)



Superannuation fund assets by fund type ($b)



LECTURE 2: INVESTMENT DECISIONS UNDER UNCERTAINTY

HOW DO WE MODEL INVESTOR PREFERENCES?

- No uncertainty = determine how much we want to consume now, and how much we want to consume later
- **Risk-free assets:**
  - Return is certain across all possible states of the world
  - Choice is between consumption NOW vs. LATER
- **Risky assets:**
  - Return is not certain across all possible states of the world
  - Range of possible future cash flows will impact on wealth
    - *If I consume today, how much will be left to spend in the future?*

UTILITY ANALYSIS

- ***Utility functions:*** provide a means to rank alternatives by scoring portfolios on the basis of their expected return + risk
  - *How does one choose the amount to invest in risky assets?* → **what is their value?**
  - E.g. How are points awarded for results in sporting matches?
    - AFL: 4 points win, 2 points draw, 0 loss
    - $\sum_W U(W)N(W)$
      - Let W be the result (win, draw, loss)
      - U(W) = points received for the result
      - N(W) = number of results of type W
- ***Expected utility theorem:*** investors choose among alternatives to maximise their expected utility
  - Use proportions P(W) instead of number N(W) – won't affect choices
    - $P(W) = \frac{N(W)}{T}$
  - $E[U(W)] = \sum_W U(W)P(W)$
    - E(U) = expectation of U (utility)
    - Weighting function is a utility function U(W)
    - P(W) = a probability
  - E.g. Consider West Ham United's standing in the English Premier League as at 26th Feb 2019.
    - Proportion calculation: $P(W) = \frac{10}{27} = 0.37$
    - Expected outcome calculation: $E(U) = points * proportion = \ 3 * 0.37 = 1.11$
      - So West Ham would expect to be awarded 1.33 points every time they play a game, when in reality it will be either 3, 1, or 0 depending on result of game

		Win	Draw	Loss	Sum
Points awarded	U(W)	3	1	0	
Number	N(W)	10	6	11	27
Sum[U(W)N(W)]		30	6	0	36
Proportion	P(W)	0.37	0.22	0.41	1
Expected outcome	E(U)	1.11	0.22	0	1.33

ASSUMPTIONS + AXIOMS OF EXPECTED UTILITY THEORY:

- Investors are assumed to be able to rank all possible alternatives
- ***Comparability****:* an investor is able to state whether they prefer A to B, B to A or if they are indifferent between A and B
- ***Transitivity****:* if an investor prefers A to B, and B to C, then the investor prefers A to C
- ***Independence****:* a chosen ranking will hold no matter what other assets are held
  - Investor is indifferent between two certain outcomes G and H, and J is uncertain → investor is indifferent b/w
    - G with probability P and J with probability (1 – P)
    - H with probability P and J with probability (1 – P)
- ***Certainty equivalent****:* for every gamble, there is a value such that the investor will be indifferent between the gamble and a 'certainty equivalent'
  - Certainty equivalent = rate that a risk-free investment would need to offer to provide the same utility score as a risky portfolio
- **Measurable:** ranking is measurable, allowing for portfolio comparison
- It is possible to rank assets + uncertain gambles

- **CRITIQUES:**
  - Independence + the existence of complements
  - Investors may not always rank alternatives consistently
  - Ranking may not be independent of the decision environment
  - Non-satiation may not be reasonable for individuals, especially at extreme levels of consumption

- **The value of outcomes depends on the utility function of the individual**
- $E[U(W)] = P(W_1)U(W_1) + P(W_2)U(W_2) + \cdots + P(W_S)U(W_S) = \sum_{s=1}^{S} U(W_S)P(W_S)$
  - S = 1, 2, ..., S and represents states of the world → i.e. recession, boom
  - An investor prefers W_1 to W_2 if and only if:
    - $E[U(W_1)] > E[U(W_2)]$
      - U(__) is an appropriate individual-specific utility function
- E.g. Choice among assets
  - An investor has a log utility $U(W) = \ln(W)$. Which asset do they prefer?

Asset A

Outcome	Utility	Probability
4	1.39	0.15
5	1.61	0.25
8	2.08	0.50
10	2.30	0.10
E(W)	U[E(W)]	E[U(W)]
6.85	1.92	1.88

Asset B

Outcome	Utility	Probability
2	0.69	0.10
3	1.10	0.30
6	1.79	0.20
8	2.08	0.30
15	2.71	0.10
E(W)	U[E(W)]	E[U(W)]
6.2	1.82	1.65

    - Step 1: Find **U(W)** figures for each asset
      - $U(4) = \ln(4) = 1.39$
      - $U(5) = \ln(5) = 1.61$ etc...
    - Step 2: Find **E(W)** for each asset
      - $E(W) = \sum_{i=1}^{4} probability * outcome$
        - $= (0.15 * 4) + (0.25 * 5) + (0.5 * 8) + (0.1 * 10) = 6.85$
    - Step 3: Find **E[U(W)]**
      - $E[U(W_A)] = \sum_{i=1}^{4} P(W_i) * U(W_i)$
        - $= (0.15 * 1.39) + (0.25 * 1.61) + (0.5 * 2.08) + (0.1 * 2.3)$
        - $= 1.88$
    - Step 4: which E[U(W)] is greater?
      - $E[U(W_A)] > E[U(W_B)]$ → $1.88 > 1.65$
        - Therefore, asset A is preferred to asset B
    - Can also find the utility of expected wealth: $U[E(W_A)] = \ln(6.85) = 1.92$
      - Investors DO NOT make decisions based on this
      - Difference between 1.92 and 1.88 represents penalty for risk

PROPERTIES OF UTILITY FUNCTIONS

- ***Non-satiation:*** the utility of more dollars is preferred to less dollars
  - If utility increases as wealth does, the **first derivative of U(W) is POSITIVE**: $U'(W) = \frac{dU(W)}{dW} > 0$
  - Suppose there are two (certain) risk-free investments with outcomes W_1 and W_2 dollars
    - If $W_1 > W_2$ then → $U(W_1) > U(W_2)$
- **Adding a constant to a utility function or multiplying utility functions by a constant does not change rankings**
  - Same investment is selected

RELATIVE RISK AVERSION (RRA)

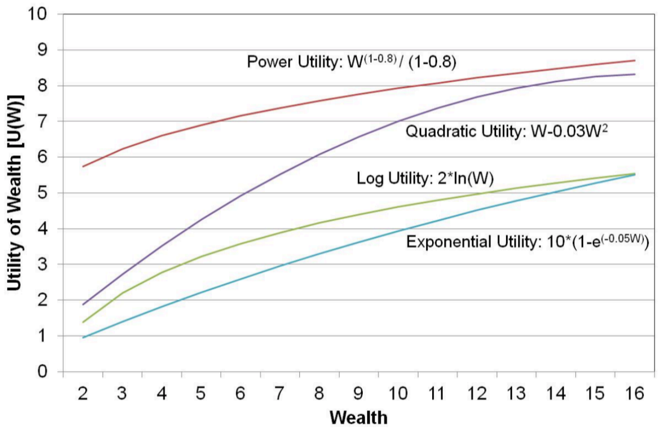
- How the **percentage of wealth** invested in risky assets changes as wealth changes
  - $RRA = R(W) = \frac{-WU''(W)}{U'(W)}$
  - $RRA = R(W) = W * ARA$
- No consensus on how relative risk aversion changes as wealth changes

CONDITION	DEFINITION	Property of A(W)	Example
Increasing RRA	Percentage invested in risky assets declines as wealth increases	R'(W) > 0	$W - bW^2$
Constant RRA	Percentage invested in risky assets is unchanged as wealth increases	R'(W) = 0	ln(W)
Decreasing RRA	Percentage invested in risky assets increases as wealth increases	R'(W) < 0	$-e^{2W-1/2}$

TYPES AND RISK AVERSION OF SELECTED UTILITY FUNCTIONS

1 – LOG UTILITY FUNCTION

- ***Log utility function***: $U(W) = \ln(W)$
  - $U'(W) = \frac{1}{W}$
  - $U''(W) = \frac{-1}{W^2}$
- **DECREASING ARA**: $A(W) = \frac{-(-\frac{1}{W^2})}{1/W} = \frac{1}{W}$
  - $A'^{(W)} = -\frac{1}{W^2} < 0$
- **CONSTANT RRA:** $R(W) = 1$
  - $R'(W) = 0$



2 – QUADRATIC UTILITY FUNCTION

- ***Quadratic utility function***: $U(W) = W - cW^2$
  - $U'(W) = 1 - 2cW$
  - $U''(W) = -2c$
- **INCREASING ARA**: $A(W) = \frac{=U''(W)}{U'(W)} = \frac{-(-2c)}{1-2cW} = 2c(1-2cW)^{-1} = \frac{2c}{(1-2cW)}$
  - $A'(W) = -2c(1-2cW)^{-2} * -2c = \frac{4c^2}{(1-2cW)^{-2}} > 0$
- **INCREASING RRA**: $(W) = \frac{-WU''(W)}{U'(W)} = \frac{2cW}{1-2cW}$
  - $R'(W) = \frac{2cW}{(1-2cW)} > 0$

3 – EXPONENTIAL UTILITY FUNCTION

- ***Exponential utility***: $U(W) = 1 - e^{-\gamma W}$
  - γ is the risk aversion coefficient
- **CONSTANT ARA**
- **INCREASING RRA**

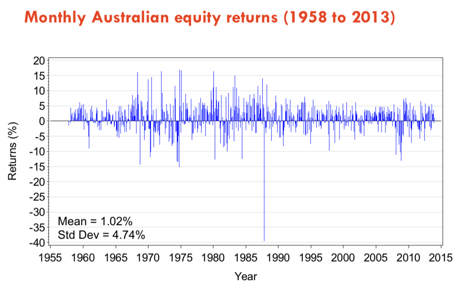
4 – POWER UTILITY FUNCTION

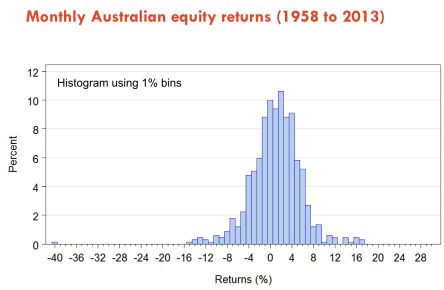
- ***Power utility***: $U(W) = \frac{W^{1-A}}{1-A}$
  - A > 0

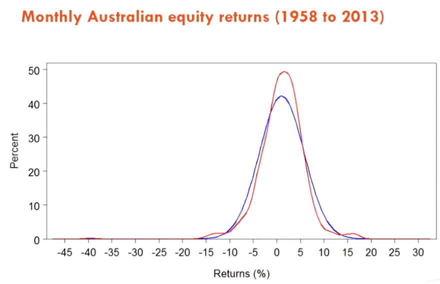
FROM WEALTH TO MEAN-VARIANCE

INVESTOR PREFERENCES

- *When is the use of expected return and standard deviation appropriate to approximate investor preferences?*
  - If **distribution of expected returns is normal**
    - Investments can be ranked according to risk and return
    - Only need mean and variance to plot returns → don't need skewness or kurtosis
  - When **utility functions are quadratic**
    - Expected utility is determined by expected wealth and standard deviation of expected wealth
- LEFT: Oct 1987 outlier → great depression
- MIDDLE: at first glance scores appear fairly normally distributed
- RIGHT: red is actual returns, blue line is the normal distribution
  - Fat tails, too peaked in centre
  - Shapiro-Wilk test indicates it is non-normal

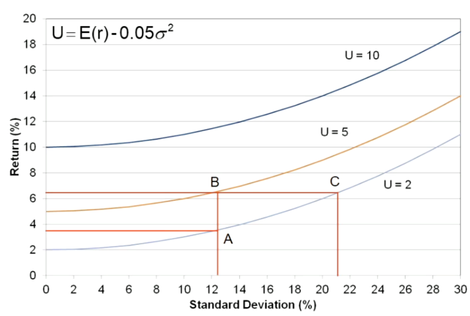






MEAN VARIANCE PREFERENCES

- Approximating investor preferences using expected return and standard deviation from quadratic utility
  - $U(W) = \mathrm{W} - cW^2$
    - Take expectations: $U[E(W)] = E(\mathrm{W}) - \mathrm{c}E(W^2)$
    - Note that: $var(W) = \mathrm{E}(W^2) - \mathrm{E(W)}^2 \rightarrow \mathrm{E}(W^2) = var(W) + \mathrm{E(W)}^2$
    - Substitute into expected utility function: $E[U(W)] = E(\mathrm{W}) - \mathrm{cE(W)}^2 - c\sigma_w^2$
- Define expected wealth as one plus the expected return
  - $E(W) = \mathrm{E}(1+\mathrm{r})$
    - $E[U(1+r) = \mathrm{E}(1+\mathrm{r}) - \mathrm{cE}(1+\mathrm{r})^2 - c\sigma_{1+r}^2$
- **Problems with quadratic utility**
  - Implies investors become satiated = beyond a point, increase in wealth reduces utility
    - People will always prefer more to less
  - Implies increasing absolute risk aversion such that risky assets are inferior
    - Risky assets should be normal goods
- Let us simplify the quadratic utility function to just mean and variance:
  - $U = E(r) - \frac{1}{2}\mathrm{A}\sigma^2$
    - U = utility derived from an investment with a particular expected return and variance (risk)
    - A = risk aversion parameter
      - A = 0 is a risk neutral investor
      - Rational investor = risk averse → A > 0
      - Risk seeking investor → A < 0
  - GRAPH:
    - Y-axis plots utility:
      - A gives utility of 2, B gives utility of 5, C gives utility of 2
    - B is preferred to C as it gives same return with less risk
    - A and C lie on same indifference curve = both give return of 2
    - Maximise utility by going as far NW as possible



- Australian fund managers are often benchmarked to ASX200 index → i.e. universe of stocks to choose from
  - To construct investment portfolios on the opportunity set requires:
    - Expected returns: 200
    - Variances: 200
    - Covariances: 19,900 = (200^2 – 200)/2
    - Total = 20,300
  - **SO, BENEFIT OF THE SINGLE INDEX MODEL IN WEEK 4**

GAINS FROM DIVERSIFICATION

- Degree to which a two-security portfolio reduces variance of returns depends on degree of correlation between returns of the securities
  - Diversification reduces risk by **offsetting firm specific risk** among assets → correlation < 1 among asset returns
  - Stabilise rate of return so variance decreases without reducing portfolio E(R_P)
- In a portfolio context, variance has two components:
  - ***Systematic risk:*** cannot be diversified away
    - Macro events affect all securities = common factor
  - ***Idiosyncratic risk:*** firm-specific risk removed via diversification
- **As more assets are added to portfolio, portfolio variance declines on avg.**
  - $\sigma_p^2 = \left(\frac{1}{N}\right)\bar{\sigma}_i^2 + \left(\frac{1-N}{N}\right)\bar{\sigma}_{ij}^2$
  - EXAMPLE RIGHT:
    - As N increases, 1/N will disappear → specific risk disappears
    - (N – 1)/N approaches 1 as N increases → covariance risk remains

$$\sigma_p^2 = \sum_{i=1}^{N} x_i^2 \sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ i \neq j}}^{N} x_i x_j \sigma_{ij}$$

$$X_i = \frac{1}{N}$$

$$\sigma_p^2 = \sum \frac{1}{N}\frac{1}{N}\sigma_i^2 + \sum_{i \neq j}\sum \sigma_{ij}\frac{1}{N^2}$$

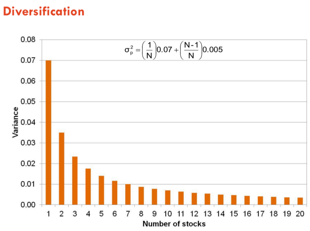
$$= \frac{1}{N}\bar{\sigma}_i^2 + \frac{1}{N^2}\sum_{i \neq j}\sum \sigma_{ij}$$

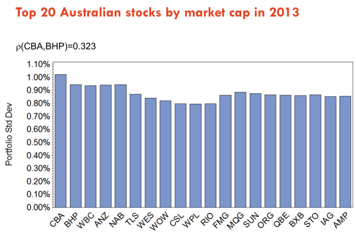
$$= \frac{1}{N}\bar{\sigma}_i^2 + (N^2 - N)\frac{1}{N^2}\underbrace{\frac{1}{(N^2 - N)}\sum_{i \neq j}\sum \sigma_{ij}}_{\bar{\sigma}_{ij}}$$

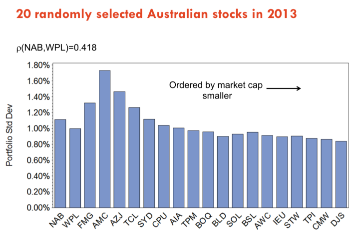
$$= \frac{1}{N}\bar{\sigma}_i^2 + (N^2 - N)\frac{1}{N^2}\bar{\sigma}_{ij}$$

$$= \frac{1}{N}\bar{\sigma}_i^2 + \frac{N-1}{N}\bar{\sigma}_{ij}$$

- *Does portfolio variance always decline as more stocks are added?*
  - LEFT: theory
  - SECOND LEFT: application → portfolio with 5% in each stock
  - FIRST RIGHT: real world application → diversification benefits depend on construction of portfolio
  - RIGHT: correlation and covariance matrixes
    - Variance changes depends what you add to your portfolio
    - FMG has high variance, AMC has huge variance → adding back all idiosyncratic risk







Correlation	NAB	WPL	FMG	AMC	AZJ
NAB	1.000	0.418	0.220	0.087	0.346
WPL	0.418	1.000	0.334	0.084	0.349
FMG	0.220	0.334	1.000	0.085	0.287
AMC	0.087	0.084	0.085	1.000	0.043
AZJ	0.346	0.349	0.287	0.043	1.000

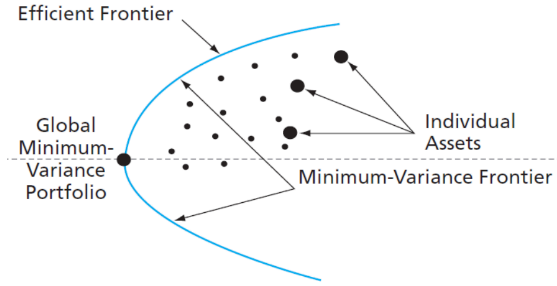
Covariance	NAB	WPL	FMG	AMC	AZJ
NAB	0.0123	0.0058	0.0072	0.0053	0.0044
WPL	0.0058	0.0160	0.0119	0.0059	0.0050
FMG	0.0072	0.0119	0.0785	0.0124	0.0090
AMC	0.0053	0.0059	0.0124	0.2748	0.0028
AZJ	0.0044	0.0050	0.0090	0.0028	0.0129

GENERATING THE OPPORTUNITY SET

- ***Efficient portfolio:*** no other portfolio will have the same expected return + lower variance of returns
  - Find investment proportions (x_1, x_2, … ,x_n) which minimise variance for given expected return subject to the constraints on proportions
    - Minimise: $\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sigma_p^2$
    - Subject to:
      - $\sum_{i=1}^{N} x_i E(R_i) = E(R_p)$
      - $\sum_{i=1}^{N} x_i = 1$

LAGRANGE MINIMISATION

- We can use ***Lagrange minimisation*** to find the optimal weights
  - $c = \sum_{i=1}^{n} x_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} x_i x_j \rho_{ij}\,\sigma_i\sigma_j + \lambda_1[1 - \sum_{i=1}^{n} x_i] + \lambda_2[E(R_P) - \sum_{i=1} x_i E(R_i)_i]$
    - Where λ_1 and λ_2 are Lagrange multipliers
    - c is a function of *(n + 2)* unknowns
- **USE SOLVER IN EXCEL**
- ***Minimum-variance set:*** result of minimising portfolio variance at each expected level of portfolio return
  - Includes efficient and inefficient sets
  - Investors only care about those on ***efficient frontier*** as they maximise value (NW decision rule)
  - GRAPH RIGHT→



- E.g. A pension fund manager is considering two mutual funds. The first is a stock fund, the second a long-term government and corporate bond fund. The risk-free rate is 8%. Correlation between fund returns is 0.10
  - What are the investment proportions in the minimum-variance portfolio of the two risky funds, and what is the expected value and standard deviation of its rate of return?

	Expected Return	Standard Deviation
Stock Fund	20%	30%
Bond Fund	12%	15%

$\sigma_S^2 = 900$

$\sigma_B^2 = 225$

$\sigma_{SB} = \rho_{SB}\sigma_S\sigma_B = 0.1 \times 30 \times 15 = 45$

$x_S = \dfrac{\sigma_B^2 - \sigma_{SB}}{\sigma_S^2 + \sigma_B^2 - 2\sigma_{SB}} = \dfrac{225 - 45}{900 + 225 - 90}$

$= 0.174$

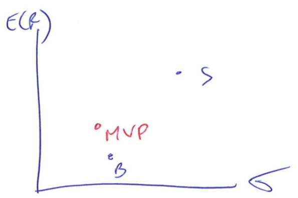
$x_B = 1 - x_S = 0.826$

$E(R_P) = 0.174(20) + 0.826(12)$

$= 13.39\%$

$\sigma_P = \sqrt{(0.174^2 \times 900) + (0.826^2 \times 225) + 2(0.174 \times 0.826 \times 45)}$

$= 13.92\%$

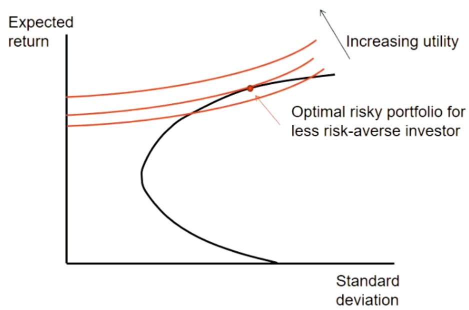


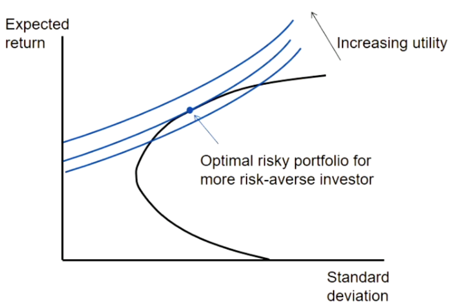
    - CHECK: Bond fund is less risky = should have higher weighting
    - CHECK: Final portfolio SD should be lower than SD of individual funds → i.e. 13.92% < 15%

CAPITAL ALLOCATION BETWEEN RISK-FREE AND RISKY ASSETS

OPTIMAL RISKY PORTFOLIO

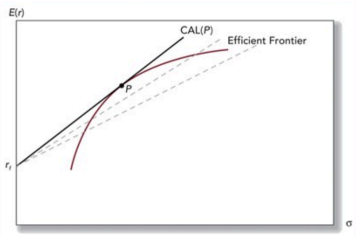
- ***Mean-variance criterion:*** The risky portfolio investors select is based upon their **preferences**
  - Depends on the investors utility function in expected return-standard deviation space
  - $E(r_A) \geq E(r_B)$ and $\sigma_A \leq \sigma_B$
- Investors select portfolios on efficient frontier → provide **maximum expected return E(R) for a given level of risk**



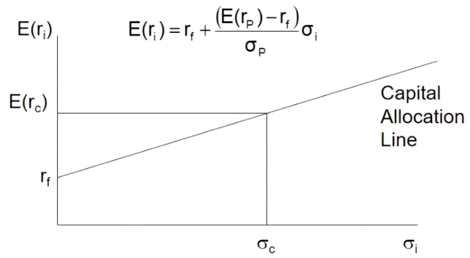


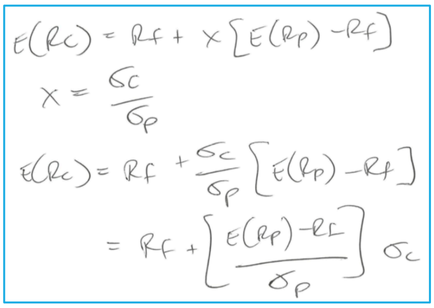
A RISKY AND **RISK-FREE** ASSET

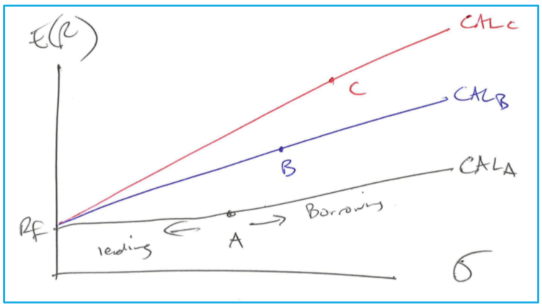
- *What portfolios are available by altering the amount invested in the risky portfolio and the risk-free asset?*
- **Expected return** for combined (or balanced) portfolio is:
  - $E(R_C) = (1-x)E(R_f) + xE(R_p) = (1-x)R_f + xE(R_p)$
    - $= R_f + x[E(R_p) - R_f]$



- **Standard deviation**:
  - $Var(R_c) = (1-x)^2Var(R_f) + x^2Var(R_p) + 2(1-x)xCov(R_p, R_f)$
    - Risk-free asset has no standard deviation = disappears
    - $= x^2Var(R_p)$
      - $\sigma_c = x\sigma_p$
  - Straight line in expected return-standard deviation space
  - ***Capital Allocation Line (CAL)*** → RIGHT
    - Sharpe Ratio = gradient of CAL
- GRAPH: Sharpe Ratio maximisation = line with the highest gradient
  - Choice between borrowing and lending



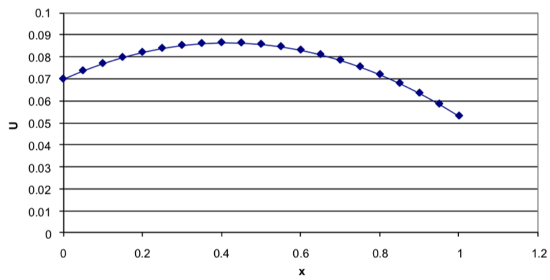




- **The Sharpe Ratio:** represents the reward-to-variability ratio → i.e. the additional reward (expected return) an investor receives for each additional unit of risk they take on
  - $Sharpe\ Ratio = \frac{E(r_p) - r_f}{\sigma_P}$
  - Gradient of CAL
  - With different borrowing and investing rates, Sharpe Ratio falls when one invests more than 100% of their wealth in the risky fund

RISK AVERSION AND ASSET ALLOCATION

- Combine risk-return asset allocation decisions with utility function to determine where along the CAL an investor is likely to select
  - Investor is described by their risk aversion coefficient



- GRAPH: there is a point at which utility is maximised
  - Mathematically found by standard optimisation techniques
    - $maxU = E[r_c] - \frac{1}{2}A\sigma_c^2$
      - $= r_f + x(E[r_p - r_f]) - \frac{1}{2}Ax^2\sigma_P^2$
    - To solve, differentiate with respect to x and set equal to 0
      - $\frac{dU}{dx} = E(r_p) - r_f - Ax\sigma_P^2 = 0$
        - $x = \frac{E(r_p) - r_f}{A\sigma_P^2}$
  - As A increases, x decreases and vice versa as required