Contents

Topic 2: Constrained Optimisation

Consumer maximisation with negative utility functions

1) Contrast to ECON20001, the optimal bundle is not always where $MRS = RP$. E.g., if an agent's utility function always yields negative utility, the bundle $(1,1)$ will yield optimal utility, irrespective of income, as it yields the least negative utility. This is because we previously assumed an agent must spend all their income, but to maximise utility, sometimes they do not.[12]

   A budget constraint with these (non-monotonic) preferences must resemble: $p_1 q_1 + p_2 q_2 \leq Y$, as opposed to (monotonic preferences): $p_1 q_1 + p_2 q_2 = Y$. Monotonic preferences allow us to assume the agent will spend all their income, i.e., it allows us to replace $\leq$ with $=$.

Solving maximisation problems:[13]

1) If **perfect substitutes**: corner solution: $\max u\left(\frac{Y}{p_1}, \frac{Y}{p_2}\right)$.
2) If **perfect complements**: e.g. $u(\boldsymbol{q}) = \min(2q_1, q_2)$
   Agent must consume in fixed proportions: $2q_1 = q_2$.[14] This says each unit of q_2 must be consumed with 2 units of q_1, to maximise utility. I.e., this is the *utility maximising condition*.
   a) To maximise utility, you would then plug $2q_1 = q_2$ into the budget constraint to solve for q_1^*, and then plug that back into $2q_1 = q_2$ to solve for q_2^*.
   b) Maximum utility is then: $\min(2q_1^*, q_2^*)$.[15]
3) If **monotonic***:* use Lagrange method: Where the Lagrangian is defined as the objective utility function + the Lagrangian Multiplier × the constraint function:

$$\max_{q1,q2,\lambda} \mathcal{L} = u(q_1, q_2) + \lambda[Y - p_1 q_1 - p_2 q_2]$$

   a) <u>Step 1</u>: Take FOC for each choice variable (q_1, q_2, λ):[16]

$$\frac{\partial \mathcal{L}}{\partial q_1} = 0, \qquad \frac{\partial \mathcal{L}}{\partial q_2} = 0, \qquad \frac{\partial \mathcal{L}}{\partial \lambda} = 0$$

   b) <u>Step 2</u>: Eliminate λ:

$$\frac{\frac{\partial \mathcal{L}}{\partial q_1}}{\frac{\partial \mathcal{L}}{\partial q_2}}$$ [17]

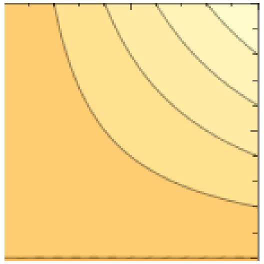
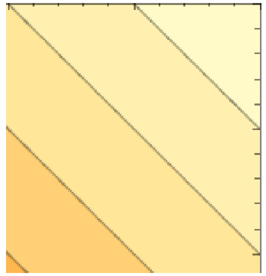
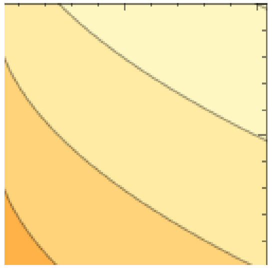
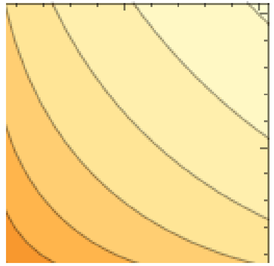
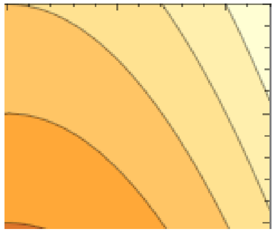
   c) <u>Step 3</u>: Rearrange to isolate one choice variable.
   d) <u>Step 4</u>: Plug into $\frac{\partial \mathcal{L}}{\partial \lambda}$ and solve for q_i^*.
   e) <u>Step 5</u>: Plug into step 3 to solve for q_j^*.

$$(q_1^*, q_2^*) = [q_1^*(\boldsymbol{p}, Y) + q_2^*(\boldsymbol{p}, Y)] = Marshallian\ Demand\ equations$$ [18]

4) If **non-monotonic***:* use Lagrange method.
   a) If $q_i^* < 0$, this is an answer that doesn't make sense, as we can't consume negative quantities:
      i) Answer will be a corner solution of $q_i^* = 0,\ q_j^* = \frac{Y}{p_j}$:

[12] This is a violation of monotonicity, as 'more is better' is not true in this instance; 'less is better'.
[13] Always check answer for optimal bundle makes sense – e.g. of non-sensical answer is $q_1^* < 0$ – see below to answer this type of question. Another non-sensical answer is when q_1 or q_2 disappear when taking FOC – this is similar to perfect substitutes.
[14] Just equate whatever is in the min function's parentheses.
[15] Utility is the smaller number of the two values.
[16] $\frac{\partial u}{\partial q_1}$ and $\frac{\partial u}{\partial q_2}$ are MU_1 and MU_2, as we are taking the change in utility given a change in quantity consumed, i.e., the marginal benefit from an increase in consumption.
[17] After eliminating λ and moving prices to other side, we get: $\frac{MU_1}{MU_2} = MRS_{1,2} = RP = \frac{p_1}{p_2}$.
[18] What an agent's optimal bundle is for a given set of prices $\boldsymbol{p}$ and income Y.

Graphed functions

Function	Graph	Function	Graph
Convex: $Any\ multiplicative\ function$, exponents irrelevant, additional linear terms irrelevant. $u = x + y + xy$ *Maximising:* Corner solution if a good <0 or > Y allows. Standard Lagrangian if good ≥0 and Y allows.		*Perfect substitutes:* $Any\ linear\ function$ (no exponents or multiplicative terms (for different variables, multiplicative terms with constants still linear), number of terms irrelevant, just all must be linear – if one term is multiplicative, will be convex. $u = 5x + y - 3$ *Maximising:* Corner solution $\max\left\{u\left(\frac{Y}{p_1}\right), u\left(\frac{Y}{p_2}\right)\right\}$	
Quasi-linear (sqrt): $u = \sqrt{x} + y$ *Maximising:* Corner solution if a good < 0 or $> Y$ allows. Standard Lagrangian if good ≥ 0 and Y allows, but with one less step as one less choice variable.		*Hybrid*: $u = \sqrt{x} + \sqrt{y}$	
Quasi-linear (sq): $u = x^2 + y$ *Maximising:* Corner solution if a good < 0 or $> Y$ allows. Standard Lagrangian if good ≥ 0 and Y allows, but with one less step as one less choice variable.		Hybrid: strictly concave $u = x^2 + y^2$ *Maximising:* A combination of goods is worse, therefore it is a corner solution: $\max\left\{u\left(\frac{Y}{p_1}\right), u\left(\frac{Y}{p_2}\right)\right\}$ (can see how on same BC, a combination yields lower indifference curve) and FOCs and Lagrangian will not help to solve.