FINM2002:

- **Derivative;** A derivative is an instrument whose value depends on the values of other more basic underlying variables
- **Futures contract;** is an agreement to buy or sell an asset at a certain time in the future for a certain price. It is a obligation so must be executed unlike options which have a right but no obligation.
  - Long futures position – the buyer of a futures contract.
  - Short futures position – the seller of a futures contract.
  - Futures price – the price at which the contract is agreed upon where the price is for delivery at some time in the future.
  - Spot price – the spot price if for immediate, or almost immediate, delivery so different to the futures price. The spot price may be higher or lower than the futures price but the two will converge as the delivery date nears. This has to happen as if it does not there is a clear arbitrage opportunity.
- **Hedge ratio;** is the ratio of the size of the position taken in futures contracts to the size of the exposure.
- **Calculating the minimum variance hedge ratio (optimal ratio);** the ratio shows which hedge ratio provides the minimum variance.
  - $h^* = \rho \frac{\sigma_S}{\sigma_F}$
  - σ_S is the standard deviation of the change in the spot price.
  - σ_F is the standard deviation of the change in the futures price.
  - ρ is the coefficient of correlation between the two standard deviations.
  - h^* is the ratio of the average change in the spot price for a particular change in the futures price.
- **Consumption asset;** an asset that is held primarily for consumption and not usually for investment purposes.
  - $F_0 = S_0(1 + r + q)^T$ if LHS is not equal to the RHS there is a arbitrage oppurtunity
  - $F_0 > S_0(1 + r + q)^T$
    - Enter into a short futures contract
    - Borrowing at risk-free rate to purchase the underlying asset
    - Paying the storage cost to hold the underlying asset to maturity.
  - $F_0 < S_0(1 + r + q)^T$
    - Enter into a long futures contract
    - Short sell underlying asset and use proceeds to invest at risk free rate.
- **Measuring Interest rates;**
  - $Future\ Value\ (Continuous\ Compounding) = PVe^{rt}$
  - $Present\ Value\ (Continuous\ Compounding) = FVe^{-rt}$
  - $Converting\ annual\ nominal\ rate\ to\ continuously\ compounded\ rate = R_c = m \times \ln(1 + \frac{R_m}{m})$
- **Spreads;** a spreading strategy involves taking a position in two or more options of the same type (i.e. two or more calls or two or more puts).
  - Bull spreads – created by buying a call option on a stock with a certain price and selling a call option on the same stock with a higher strike price. The investor believes the spot price will rise moderately. Both options have the same expiration day. Payoff;

Stock price range	Payoff from long call	Payoff from short call	Total payoff
$S_T \le K_1$	0	0	0
$K_1 < S_T < K_2$	$S_T - K_1$	0	$S_T - K_1$
$S_T \ge K_2$	$S_T - K_1$	$K_2 - S_T$	$K_2 - K_1$

    - S_T is the stock price on the expiration day
    - K_1 is the strike price of the call option bought
    - K_2 is the strike price of the call option sold
  - Bear spreads – created by buying a put with one strike price and selling a put with another strike price which is lower than the strike price of the put bought. Payoff;

Stock price range	Payoff from long put	Payoff from short put	Total payoff
$S_T \le K_1$	$K_2 - S_T$	$S_T - K_1$	$K_2 - K_1$
$K_1 < S_T < K_2$	$K_2 - S_T$	0	$K_2 - S_T$
$S_T \ge K_2$	0	0	0

    - S_T is the stock price on the expiration day
    - K_1 is the strike price of the call option sold
    - K_2 is the strike price of the call option bought
    - $where\ K_1 < K_2$
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- **General binomial model;** using a one stock and option approach.
  - If there is an up movement in the stock price, the value of the portfolio at the end of the life of the option is;
    - $S_0 u\Delta - f_u$
  - If there is a down movement in the stock price, the value becomes;
    - $S_0 d\Delta - f_d$
  - The two are equal when;
    - $\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$
  - If we denote the risk-free interest rate by r, the present value of the portfolio is;
    - $(S_0 u\Delta - f_u)e^{-rT}$
  - The cost of setting up the portfolio is;
    - $S_0\Delta - f$
  - It follows that;
    - $f = S_0\Delta(1 - ue^{-rT}) + f_u e^{-rT}$
  - Substituting from above (3) we have;
    - $f = S_0\left(\frac{f_u - f_d}{S_0 u - S_0 d}\right)(1 - ue^{-rT}) + f_u e^{-rT}$
    - $f = \frac{f_u(1 - de^{-rT}) + f_d(ue^{-rT} - 1)}{u - d}$
    - $f = e^{-rT}[pf_u + (1 - p)f_d]$
      - $where\ p = \frac{e^{rT} - d}{u - d}$
- **Risk neutral valuation;** we can assume investors are risk neutral when valuing a derivative. Risk-neutral investors do not increase the expected return they require from a investment to compensate for increased risk. A risk neutral world has two features that simplify the pricing of derivatives;
  - The expected return on a stock (or any other investment) is the risk free rate.
  - The discount rate used for the expected payoff on an option (or any other derivative) is the risk-free rate.
- **General two step binomial model;**
  - Because the length of a time step is now Δt rather than T, equations from (7) above become;
    - $f = e^{-r\Delta T}[pf_u + (1 - p)f_d]$
      - $where\ p = \frac{e^{r\Delta T} - d}{u - d}$
  - Repeated application of the equation above (1) gives the following;
    - $f_u = e^{-r\Delta T}[pf_{uu} + (1 - p)f_{ud}]$
    - $f_d = e^{-r\Delta T}[pf_{ud} + (1 - p)f_{dd}]$
    - $f = e^{-r\Delta T}[pf_u + (1 - p)f_d]$
  - Substituting the first two from (2) into the last from (2) we get;
    - $f = e^{-2r\Delta T}[p^2 f_{uu} + 2p(1 - p)f_{ud} + (1 - p)^2 f_{dd}]$
    - The variables p^2, $2p(1 - p)$ and $(1 - p)^2$ are the probabilities that the upper, middle and lower final nodes will be reached.
- **Options on stocks paying known dividend yields;**
  - Lower bounds for options with known dividend yields (if currencies replace qt with rf in formulas)–
    - Lower bound for the European call option;
      - $c \geq S_0 e^{-qT} - Ke^{-rT}$
    - Lower bound for the European put option;
      - $p \geq Ke^{-rT} - S_0 e^{-qT}$
  - Put-Call Parity –
    - $c + Ke^{-rT} = p + S_0 e^{-qT}$
    - For American options, the put-call parity relationship is;
      - $S_0 e^{-qT} - K \leq C - P \leq S_0 - Ke^{-rT}$
  - Pricing formulas in Black-Scholes-Merton model (with dividend yield) –
    - $c = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2)$
    - $p = Ke^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$
    - Since;
      - $ln\frac{S_0 e^{-qT}}{K} = ln\frac{S_0}{K} - qT$
    - Then it implies that;
      - $d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$
      - $d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$