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Articles

Signals

Signal (electronics)

A **signal** is any stream of quantities in time or spatial sequence. *Signals* categorizes to the fields of communications, signal processing, and to electrical engineering more generally.

Signals may contain and transport information in coded form like modulation. In that case the actual quantities are finally not used, but are decoded in a detector or demodulator.

In the physical world, any quantity measurable through time or over space can be taken as a signal. Within a complex society, any set of human information or machine data can also be taken as a signal. Such information or machine data (for example, the dots on a screen, the ink making up text on a paper page, or the words now flowing into the reader's mind) must all be part of systems existing in the physical world – either living or non-living.

Despite the complexity of such systems, their outputs and inputs can often be represented as simple quantities measurable through time or across space. In the latter half of the 20th century, electrical engineering itself separated into several disciplines, specializing in the design and analysis of physical signals and systems, on the one hand, and in the functional behavior and conceptual structure of the complex human and machine systems, on the other. These engineering disciplines have led the way in the design, study, and implementation of systems that take advantage of signals as simple measurable quantities in order to facilitate the transmission, storage, and manipulation of information.

Some definitions

Definitions specific to subfields are common. For example, in information theory, a *signal* is a codified message, that is, the sequence of states in a communication channel that encodes a message.

In the context of signal processing, arbitrary binary data streams are not considered as signals, but only analog and digital signals that are representations of analog physical quantities.

In a *communication system*, a *transmitter* encodes a *message* into a signal, which is carried to a *receiver* by the communications *channel*. For example, the words "Mary had a little lamb" might be the message spoken into a telephone. The telephone transmitter converts the sounds into an electrical voltage signal. The signal is transmitted to the receiving telephone by wires; and at the receiver it is reconverted into sounds.

In telephone networks, signalling, for example common-channel signaling, refers to phone number and other digital control information rather than the actual voice signal.

Signals can be categorized in various ways. The most common distinction is between discrete and continuous spaces that the functions are defined over, for example discrete and continuous time domains. Discrete-time signals are often referred to as *time series* in other fields. Continuous-time signals are often referred to as *continuous signals* even when the signal functions are not continuous; an example is a square-wave signal.

A second important distinction is between discrete-valued and continuous-valued. Digital signals are sometimes defined as discrete-valued sequencies of quantified values, that may or may not be derived from an underlying continuous-valued physical process. In other contexts, digital signals are defined as the continuous-time waveform signals in a digital system, representing a bit-stream. In the first case, a signal that is generated by means of a digital modulation method is considered as converted to an analog signal, while it is considered as a digital signal in the second case.signal transfers information

Discrete-time and continuous-time signals

If for a signal, the quantities are defined only on a discrete set of times, we call it a discrete-time signal. In other words, a discrete-time real (or complex) signal can be seen as a function from (a subset of) the set of integers to the set of real (or complex) numbers.

A continuous-time real (or complex) signal is any real-valued (or complex-valued) function which is defined for all time *t* in an interval, most commonly an infinite interval.

Analog and digital signals

Less formally than the theoretical distinctions mentioned above, two main types of signals encountered in practice are *analog* and *digital*. In short, the difference between them is that digital signals are *discrete* and *quantized*, as defined below, while analog signals possess neither property.

Discretization

One of the fundamental distinctions between different types of signals is between continuous and discrete time. In the mathematical abstraction, the domain of a continuous-time (CT) signal is the set of real numbers (or some interval thereof), whereas the domain of a discrete-time (DT) signal is the set of integers (or some interval). What these integers represent depends on the nature of the signal.

DT signals often arise via sampling of CT signaexample, consists of a continually fluctuating voltage on a line that can be digitized by an ADC circuit, wherein the circuit will read the voltage level on the line, say, every 50 microseconds. The resulting stream of numbers is stored as digital data on a discrete-time signal. Computers and other digital devices are restricted to discrete time.

Quantization

If a signal is to be represented as a sequence of numbers, it is impossible to maintain arbitrarily high precision - each number in the sequence must have a finite number of digits. As a result, the values of such a signal are restricted to belong to a finite set; in other words, it is quantized.

Examples of signals

- *Motion*. The motion of a particle through some space can be considered to be a signal, or can be represented by a signal. The domain of a motion signal is one-dimensional (time), and the range is generally three-dimensional. Position is thus a 3-vector signal; position and orientation is a 6-vector signal.
- *Sound*. Since a sound is a vibration of a medium (such as air), a sound signal associates a pressure value to every value of time and three space coordinates. A microphone converts sound pressure at some place to just a function of time, generating a voltage signal as an analog of the sound signal. Sound signals can be sampled to on a discrete set of time points; for example, compact discs (CDs) contain discrete signals representing sound, recorded at 44,100 samples per second; each sample contains data for a left and right channel, which may be considered to be a 2-vector signal (since CDs are recorded in stereo).
- *Images*. A picture or image consists of a brightness or color signal, a function of a two-dimensional location. A 2D image can have a continuous spatial domain, as in a traditional photograph or painting; or the image can be discretized in space, as in a raster scanned digital image. Color images are typically represented as a combination of images in three primary colors, so that the signal is vector-valued with dimension three.
- *Videos*. A video signal is a sequence of images. A point in a video is identified by its two-dimensional position and by the time at which it occurs, so a video signal has a three-dimensional domain. Analog video has one continuous domain dimension (across a scan line) and two discrete dimensions (frame and line).

- Biological *membrane potentials*. The value of the signal is a straightforward electric potential ("voltage"). The domain is more difficult to establish. Some cells or organelles have the same membrane potential throughout; neurons generally have different potentials at different points. These signals have very low energies, but are enough to make nervous systems work; they can be measured in aggregate by the techniques of electrophysiology.

Entropy

Another important property of a signal (actually, of a statistically defined class of signals) is its entropy or *information content*.

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- Shannon, C. E., 2005 [1948], "A Mathematical Theory of Communication," (corrected reprint [1]), accessed Dec. 15, 2005. Orig. 1948, *Bell System Technical Journal*, vol. 27, pp. 379–423, 623-656.

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Even and odd functions

In mathematics, **even functions** and **odd functions** are functions which satisfy particular symmetry relations, with respect to taking additive inverses. They are important in many areas of mathematical analysis, especially the theory of power series and Fourier series. They are named for the parity of the powers of the power functions which satisfy each condition: the function $f(x) = x^n$ is an even function if n is an even integer, and it is an odd function if n is an odd integer.

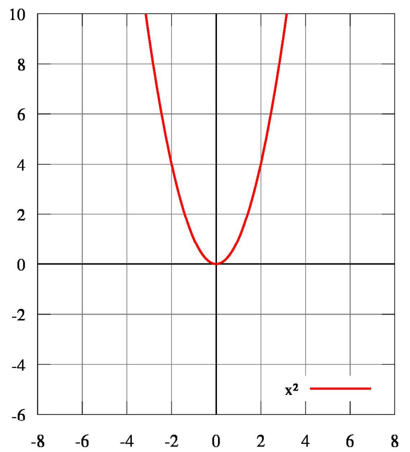
Even functions

Let $f(x)$ be a real-valued function of a real variable. Then f is **even** if the following equation holds for all x in the domain of f:

$$f(x) = f(-x).$$

Geometrically speaking, the graph face of an even function is symmetric with respect to the y-axis, meaning that its graph remains unchanged after reflection about the y-axis.

Examples of even functions are $|x|$, x^2, x^4, $\cos(x)$, and $\cosh(x)$.



$f(x) = x^2$ is an example of an even function.

Odd functions

Again, let $f(x)$ be a real-valued function of a real variable. Then f is **odd** if the following equation holds for all x in the domain of f:

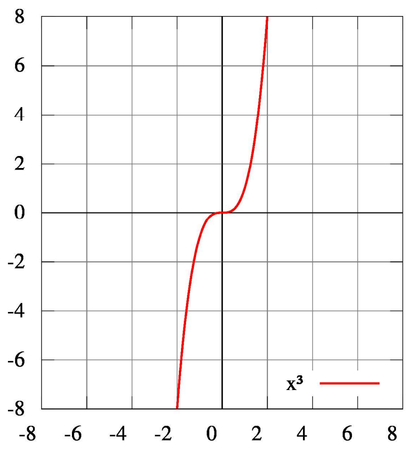
$$-f(x) = f(-x),$$

or

$$f(x) + f(-x) = 0.$$

Geometrically, the graph of an odd function has rotational symmetry with respect to the origin, meaning that its graph remains unchanged after rotation of 180 degrees about the origin.

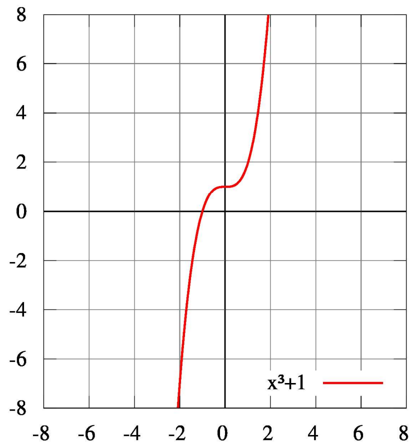
Examples of odd functions are x, x^3, $\sin(x)$, $\sinh(x)$, and $\operatorname{erf}(x)$.



$f(x) = x^3$ is an example of an odd function.

Some facts

A function's being odd or even does not imply differentiability, or even continuity. For example, the Dirichlet function is even, but is nowhere continuous. Properties involving Fourier series, Taylor series, derivatives and so on may only be used when they can be assumed to exist.



$f(x) = x^3 + 1$ is neither even nor odd.

Basic properties

- The only function which is *both* even and odd is the constant function which is equal to zero (i.e., $f(x) = 0$ for all x).
- The sum of an even and odd function is neither even nor odd, unless one of the functions is equal to zero over the given domain.
- The sum of two even functions is even, and any constant multiple of an even function is even.
- The sum of two odd functions is odd, and any constant multiple of an odd function is odd.
- The product of two even functions is an even function.
- The product of two odd functions is an even function.
- The product of an even function and an odd function is an odd function.
- The quotient of two even functions is an even function.
- The quotient of two odd functions is an even function.
- The quotient of an even function and an odd function is an odd function.
- The derivative of an even function is odd.
- The derivative of an odd function is even.
- The composition of two even functions is even, and the composition of two odd functions is odd.
- The composition of an even function and an odd function is even.
- The composition of any function with an even function is even (but not vice versa).

- The integral of an odd function from −*A* to +*A* is zero (where *A* is finite, and the function has no vertical asymptotes between −*A* and *A*).
- The integral of an even function from −*A* to +*A* is twice the integral from 0 to +*A* (where *A* is finite, and the function has no vertical asymptotes between −*A* and *A*).

Series

- The Maclaurin series of an even function includes only even powers.
- The Maclaurin series of an odd function includes only odd powers.
- The Fourier series of a periodic even function includes only cosine terms.
- The Fourier series of a periodic odd function includes only sine terms.

Algebraic structure

- Any linear combination of even functions is even, and the even functions form a vector space over the reals. Similarly, any linear combination of odd functions is odd, and the odd functions also form a vector space over the reals. In fact, the vector space of *all* real-valued functions is the direct sum of the subspaces of even and odd functions. In other words, every function *f*(*x*) can be written uniquely as the sum of an even function and an odd function:

  $$f(x) = f_{\rm e}(x) + f_{\rm o}(x)\,,$$

  where

  $$f_{\rm e}(x) = \tfrac{1}{2}[f(x) + f(-x)]$$

  is even and

  $$f_{\rm o}(x) = \tfrac{1}{2}[f(x) - f(-x)]$$

  is odd. For example, if *f* is exp, then f_e is cosh and f_o is sinh.
- The even functions form a commutative algebra over the reals. However, the odd functions do *not* form an algebra over the reals.

Harmonics

In signal processing, harmonic distortion occurs when a sine wave signal is sent through a memoryless nonlinear system, that is, a system whose output at time t only depends on the input at time t and does not depend on the input at any previous times. Such a system is described by a response function $V_{\text{out}}(t) = f(V_{\text{in}}(t))$. The type of harmonics produced depend on the response function f:[1]

- When the response function is even, the resulting signal will consist of only even harmonics of the input sine wave; $2f, 4f, 6f, \ldots$
  - The fundamental is also an odd harmonic, so will not be present.
  - A simple example is a full-wave rectifier.
- When it is odd, the resulting signal will consist of only odd harmonics of the input sine wave; $1f, 3f, 5f, \ldots$
  - The output signal will be half-wave symmetric.
  - A simple example is clipping in a symmetric push-pull amplifier.
- When it is asymmetric, the resulting signal may contain either even or odd harmonics; $1f, 2f, 3f, \ldots$
  - Simple examples are a half-wave rectifier, and clipping in an asymmetrical class A amplifier.