FINC6022

Behavioral Finance

S1 2018

Table of Contents

FINC6022 TOPIC 1: Expected Utility

"Rational" theories of finance tell us how people 'should' behave – and often do not reflect reality.

A normative theory based on rational utility maximizers cannot be construed as a superior alternative to behavioral approaches, merely because it discusses how people should behave but a not well-explained empirically.

If people do not behave this way, there are limitation to helping us understand the observed market behavior.

Neoclassical economics

1. People have rational preferences across possible outcomes
2. People maximize utility and firms maximize profits
3. People make independent decisions based on all relevant information

Expected Utility theory

First proposed by Daniel Bernoulli in 1728 in response to solving what reasonable price an individual should pay to enter a gamble.

A coin is flipped repeatedly until a head is produced; if you enter the game you receive a payoff of $\$2^n$ where n is the number of the throw producing the first head.

E(V) = ½ (2) + ¼ (4) + ⅛(8) + … = 1 + 1 + 1+ … or in general $E(V) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n 2^n$

But even though the expected value of this gamble is infinite, most people would be unwilling to pay more than a few dollars – ***St. Petersburg Paradox.***

Bernoulli chose a logarithmic utility function to explain that the expected utility of the game is indeed finite.

$E\big(U(V)\big) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \ln\left(2^n\right) \approx 1.386$. This corresponds to a certain value (Certainty Equivalent) of $e^{1.386} \approx \$4.00$

where U(x) = ln(x)

Preferences are defined over prospects, where a prospect is a list of consequences or outcomes with associated probabilities.

- Assume all consequences and probabilities are known to the investor.

In choosing among prospects, the investor can be said to confront a situation of risk – (in contrast with situations of uncertainty in which as least some of the outcomes or probabilities are unknown)

- Any prospect **q** can be represented by a probability distribution **q** = ($p_1, p_2, \ldots, p_n$) over a fixed set of pure consequences X = ($x_1, x_2, \ldots, x_n$), where p_i is the probability of x_i, $p_i \geq 0$ and $\Sigma p_i = 1$.

Reducing compound lottery

$3	-$8		-$8	$12	$3
2/3	1/3		1/4	5/12	1/3
1/2			1/2		

=

-$8	$3	$12
7/24	1/2	5/24

All the lotteries involved in a compound lottery are always assumed to be *independent* of each other and so it is easy to reduce a compound lottery to a simple lottery

Experimental evidence has suggested that people tend to prefer the compound form of the lottery (on the left, above), rather than its reduced form (on the right here). This is particularly likely when the probabilities of winning in the first part of the compound lottery are high.??

Axioms of EUT

There are four axioms of the expected utility theory that define a *rational* decision maker. They are completeness, transitivity, independence and continuity.

Completeness:
For all **q**, **r**: either $\boldsymbol{q} \gtrsim \boldsymbol{r}$ or $\boldsymbol{r} \gtrsim \boldsymbol{q}$ or $\boldsymbol{q} \backsim \boldsymbol{r}$
where $\gtrsim$ represents the relation: "**q** is (weakly) preferred to **r**".

Transitivity:
For all **q**, **r**, **s**: if$\boldsymbol{q} \gtrsim \boldsymbol{r}$ and $\boldsymbol{r} \gtrsim \boldsymbol{s}$ then $\boldsymbol{q} \gtrsim \boldsymbol{s}$

Continuity: requires that for all prospects q, r, s
- Where $\boldsymbol{q} \gtrsim \boldsymbol{r}$ and $\boldsymbol{r} \gtrsim \boldsymbol{s}$, there exists some probability p such that $\left(\boldsymbol{q}, p; \mathbf{s},\ (1-\mathrm{p})\right) \sim \boldsymbol{r}$
- that is, there is some mixture of the prospects **q** and **s** for which the investor will be indifferent to choosing prospect **r**

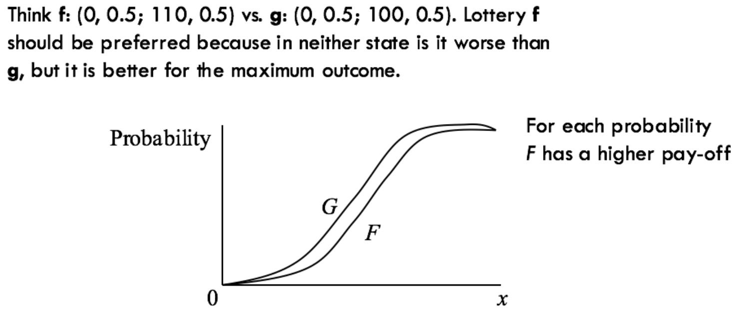
Independence:

It assumes that two gambles mixed with an irrelevant third one will maintain the same order of preference as when the two are presented independently of the third one.

EUT provides one very simple way of combining probabilities and consequences into a single "measure of value" which has a number of appealing properties.

One such property is *monotonicity:*

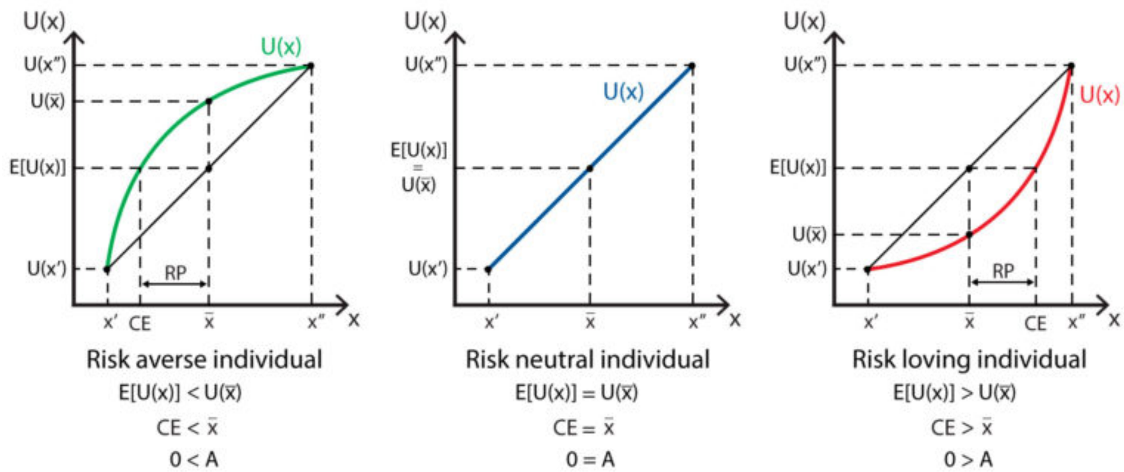
- Monotonicity is the property that stochastically dominating prospects are always preferred to prospects which they dominate.
- This is a fairly basic concept in rationality – it says that if **q** stochastically dominates **r**, then the expected value of **q** is higher than **r**.



The expected utility theory takes into account that individuals may be risk-averse, meaning that the individual would refuse a fair gamble (a fair gamble has an expected value of zero)

- The shape of the utility function also has a simple behavioural interpretation whereby concavity (convexity) of *u(.)* implies risk averse (seeking) behaviour.

- Someone with a concave utility function will always prefer a certain amount X to any risky prospect with expected value equal to X.
  - E.g. A risk-averse individual will accept less than $50 rather than take a coin toss which would yield $100 if heads and $0 otherwise.
    - If they would accept $40 for sure, rather than taking the gamble, this would be the *certainty equivalent*. We work this out by finding the certain value with the same utility as the risky gamble.
    - The difference between the expected value and the certainty equivalent is the *risk premium* from taking the gamble.



Absolute risk aversion: $Ra(x) \ = \ -u''(x)/u'(x)$

- More wealth is preferred to less, but utility grows at decreasing rate. This gives the concave shape we would normally expect in utility functions.

Relative risk aversion: $Rr(x) = -x.u''(x)/u'(x)$

- As wealth increases the level of the risk premium increases at a decreasing rate.

Constant absolute risk aversion (CARA):
- E.g. $u(x) = -e^{-\alpha x}$
- level of risk aversion does not depend on wealth, risk aversion does not change with changes in wealth

Constant relative risk aversion (CRRA):
- E.g. $u(x) = x^{1-\beta}/(1-\beta)$ if $\beta \neq 1$ and $u(x) = \log(x)$ is $\beta = 1$
- As wealth increases the level of the risk premium increases proportionally

Limitations of EUT

It is precisely the simplicity and economy of EUT that has made it such a powerful and tractable modelling tool.

- The problem, however, is with the descriptive merits of the theory
  - whether EUT provides a sufficiently accurate representation of actual choice behaviour.
  - The evidence from a large number of empirical tests has raised some real doubts

- There is now a large body of evidence indicating that actual choice behaviour may ***systematically*** violate the independence axiom.
  - Systematic violations, rather than random or idiosyncratic violations, suggest descriptive failings of Expected Utility Theory
- Two examples of such phenomena are
  - Common Consequence Effect
  - Common Ratio Effect

The **Allais paradox** is a choice problem designed by Maurice **Allais** (1953) to show an inconsistency of actual observed choices with the predictions of expected utility theory.

Example –

Problem 1: Choose between Prospect A and Prospect B.
Prospect A: $2,500 with probability 0.33
$2,400 with probability 0.66
$0 with probability 0.01
Prospect B: $2,400 with certainty
Problem 2: Choose between Prospect C and Prospect D.
Prospect C: $2,500 with probability 0.33
$0 with probability 0.67
Prospect D: $2,400 with probability 0.34
$0 with probability 0.66

It has been shown by Daniel Kahneman and Amos Tversky (1979, “Prospect Theory: An analysis of decision under risk,” *Econometrica*, 47 (2), 263-291) that more people choose B with Problem 1 and more people choose C when presented with Problem 2. These choices violate Expected Utility Theory. Why?

Individuals fail to rationally value each gamble.

In the first problem, there is an overvalue on certainty. In the second problem, there is more value placed on the higher potential upside, even though the probabilities are similar.