Revision guide for MAS10006

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Contents:

Topic 1: Limits, Continuity, Convergence, Sequences and Series

Basic Stuff

Definition of a Limit

For the limit to exist:

$$L = \lim_{x\to a+} f(x) = \lim_{x\to a-} f(x)$$

L must be finite and defined.

Note that for the limit to exist the following condition does not need to be met:

$$f(a) \neq L$$

Continuous Functions

If:

$$f(a) = L$$

Then the function is continuous at x=a.

Polynomial, exponential, logarithmic, root and hyperbolic functions are continuous for every element in their domain.

If you divide, multiply, sum or subtract two continuous functions then result will be continuous (Provided the function is defined at x=a).

Expressing Domain and Range

Some examples of where a function may be undefined include:

- Even root of a negative number
- Log of zero or a negative number
- Zero as a denominator of a fraction

We use set builder notation to express domain and range.
http://www.mathsisfun.com/sets/set-builder-notation.html

Sequences

A sequence is where we replace the continuous ‘x’ in a function with a counter integer ‘n’.

A sequence is not continuous.

The n^th element in the sequence is denoted by: a_n

Series

The sum of a sequence:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots$$

Geometric Series

$$\sum_{n=1}^{\infty} a * r^n = \frac{a}{1-r}$$

Sum converges only if $r < 1$

Harmonic P Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

- If $p \leq 1$ then the series diverges
- If $L > 1$ then the series converges.

Limit solving techniques

Continuity

If the functions are continuous at $x = a$ then you can substitute x with a and compute the limit.

Limit laws

$$\lim_{n\to\infty} r^n = 0, \quad (|r| < 1)$$

$$\lim_{n\to\infty} \frac{a^n}{n!} = 0, \quad a \in \mathbb{R}$$

$$\lim_{n\to\infty} \left(1 + \frac{a}{n}\right)^n = e^a, \quad a \in \mathbb{R}$$

$$\lim_{n\to\infty} a^{\frac{1}{n}} = 1, \quad (a > 0)$$

$$\lim_{n\to\infty} \frac{\log n}{n^p} = 0 \quad (p > 0)$$

$$\lim_{n\to\infty} \frac{n^p}{a^n} = 0, \quad (p > 0, a > 1)$$

L’Hopitals rule

If you have a limit in the form:

$$\lim_{x\to\infty} f(x) = \frac{u(x)}{v(x)}$$

…and the computed limit is in the form:

$$\frac{0}{0} \ or \ \frac{\infty}{\infty} \ or \ \frac{-\infty}{-\infty}$$

Then you can instead find the limit of:

$$\lim_{x\to\infty} f(x) = \frac{u'(x)}{v'(x)}$$

Since the derivative is an operation for continuous functions, you must substitute sequences a_n with their equivalent functions $f(x)$ before using L’Hopitals rule.

Dividing by the highest power

If you have a limit similar to:

$$\lim_{x\to\infty} f(x) = \frac{n! + a^n + 2 * n^p + 3}{5 * n! + 8 * a^n + 13 * n^p + 21}$$

You can divide everything by $n!$ and use limit laws to get $\frac{1}{5}$.

Sandwich rule

$$f(x) \geq g(x) \geq h(x)$$

So:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \lim_{x\to\infty} h(x)$$