Valuation of Bonds

Terminology

- Bond: long-term debt instruments issued by governments and corporations for the purpose of borrowing money/raising funds.
- Owners of a bond receive periodic interest payments over the life of the bond, and also get back principal amount (at maturity).
- Face Value (F): nominal amount of principal borrowed.

Value of Coupon Payments

$$C = \frac{Coupon\ rate * Face\ Value}{Number\ of\ payments\ per\ year}$$

Valuing a Bond

- **The value is equal to the total present value of all coupon payments and face value of bond**.
- The first term represents the PV of all coupon payments, and the second term represents the PV of principal amount.
- Yield to Maturity (YTM) is equal to r, and summarises its prospective return, given its market price.

$$Bond\ Price\ (PV) = \frac{C}{r}[1-(1+r)^{-N}] + \frac{F}{(1+r)^N}$$

Zero Coupon Bonds

- **No coupons are payed**
- **Only the face value is paid** at maturity
- Always sells at a discount (Price < F)
- No interest is earned; your compensation is the difference between the initial price and the face value

$$Price\ of\ a\ Zero\ Coupon\ Bond = \frac{F}{(1+YTM_n)^n}$$

Bond Prices

1. Par (If Price = Face Value):
    - The only return that investors earn is from the coupons that the bond pays, hence, <u>YTM = Coupon Rate.</u>

2. Discount (if Price < Face Value):
    - Investor earns returns both from receiving the coupons and receiving a face value that exceeds the price paid, hence, <u>Coupon rate < YTM</u>.

3. Premium (If Price > Face Value):
    - Investor earns a return from receiving the coupons but this return will be diminished by receiving a face value less than price paid, hence, <u>Coupon Rate > YTM</u>.

Inverse Relationship between Bond Prices and YTM

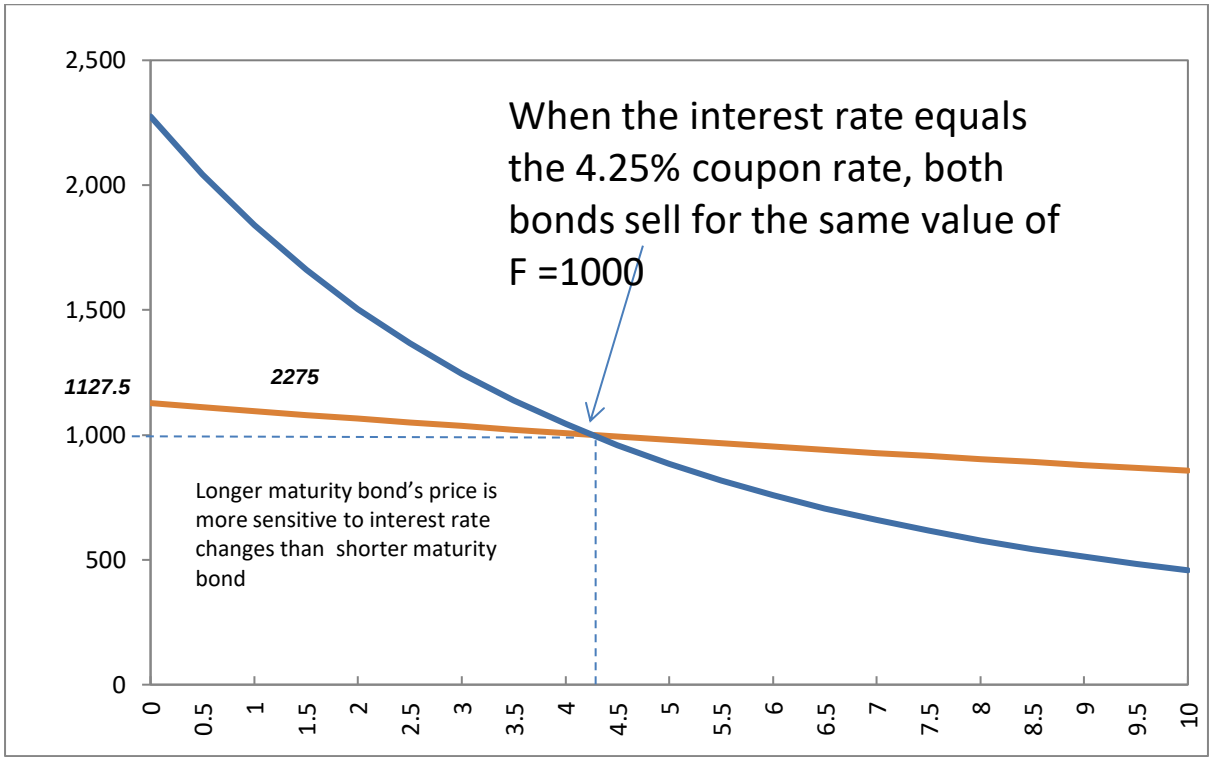
- *As YTM increases, the bond price will decrease*
- *As YTM decreases, the bond price will increase*

Maturity and Prices

- A bond with a *longer maturity* (e.g. 30 years) will be **more sensitive to interest rate changes** than a bond with a *shorter maturity* (e.g. 2 years).

Price Yield Curve

- Relationship between P & YTM:



Duration

- Maturity only measures time until final cash flow is paid, and ignores all interim cash flows.
- Hence, duration gives a more direct measure of interest sensitivity.
- Duration is a **weighted average of cash flow times** (expressed in units of time).
- The weighting coefficients are the PVs of the individual cash flows as a proportion of the total PV of all cash flows.

$$Duration\ (D) = \frac{PV(C_1).t_1 + PV(C_2).t_2 + \cdots + PV(C_t)t_n}{Total\ PV}$$

- Note: *Duration is always shorter than maturity for all bonds except for zero coupon bonds.*

Modified Duration

- Linked to the slope of the price yield curve

$$Modified\ Duration(D_m) = Volatility(\%) = \frac{Duration}{1 + YTM}$$

Debt and Interest Rates

- Nominal interest rate → the rate you actually pay when you borrow
- Real interest rate → theoretical rate paid, as determined by supply and demand

$$r_{real} = \frac{r_{nom} - i}{1 + i}$$

Where:

'i' is the expected annual inflation rate

The Risk of Default

- Corporate bonds have some level of risk → payments promised to bondholders represent the best-case scenario.
- Level of risk is dependent on the financial status of the firm, e.g. if company goes bankrupt, payments will not be received by bondholders.
- Credit risk: refers to this risk of default, and is judged by bond ratings.

Bond Ratings

Investment grade Bonds

Moody's	Standard & Poor's and Fitch
Aaa	AAA
Aa	AA
A	A
Baa	BBB

Junk Bonds

Moody's	Standard & Poor's and Fitch
Ba	BB
B	B
Caa	CCC
Ca	CC
C	C

- **Higher bond rating = lower YTM (less associated risk),**
  e.g. Johnson and Johnson bonds have a AAA rating, but YTM is only 1.27%

- **Lower bond rating = higher YTM (more associated risk),**
  e.g. Caesars Entertainment bonds have a CCC rating, however YTM = 25.7%

Sovereign Bonds and Default Risk

- Sovereign debt is generally less risky than corporate debt; inflationary policies can reduce the real value of debts.
- If a government has foreign debt, they may default, which affects bond prices and YTM.
- Having 'own' currency debt is less risky, since the government can print money to repay bonds.