

Lecture 2: Time Value of Money 1

Simple interest; Compound interest; Present Value and Future Value.

Notes:

- Readings: Ross Ch. 4 & Ch. 5 - **done**

Chapter 4 – Introduction to valuation the time value of money

Future value and compounding

Future value (FV) – the amount of money an investment will grow to over some period of time at some given interest rate.

Compounding – the process of accumulating interest in an investment over time to earn more interest.

- This means earning interest on interest → compound interest.

Future value = $PV(1 + i \times n)$

Present value and discounting

Present value (PV) – the current value of future cash flows discounted at the appropriate discount rate.

$$PV = FV / (1 + i \times n) \quad \text{or} \quad PV = \frac{FV}{(1 + i \times n)}$$

Chapter 5 – discounted cash flow

Valuing level cash flows: annuities and perpetuities

Annuity – a level stream of cash flows for a fixed time.

Annuity - Present value

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

Annuity – Future value

$$FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$$

Annuity due – an annuity for which the cash flows occur at the beginning of the period.

Perpetuities

Perpetuity – an annuity in which the cash flows continue forever.

$$PV = \frac{PMT}{i}$$

Loan types and loan amortisation

Pure discount loans

The borrower receives money today and repays a single lump sum at some time in the future.

Interest-only loans

Repayment plan that calls for the borrower to pay interest each period and to repay the entire principal (original loan amount) at some point in the future.

Amortised loans

The lender may require the borrower to repay parts of the loan amount over time.

- The process of paying off a loan by making regular principal reductions is called *amortising* the loan.

Lecture Examples

Example 2

What is the future value of \$100,000 invested for 180 days at 10% p.a. simple interest?

Solution

$$FV = PV(1 + i \times n)$$

$$\begin{aligned} FV &= 100,000(1 + 10\% \times 180/365) \\ &= 100,000(1 + 0.049315068) \\ &= \$104,931.51 \end{aligned}$$

Example 3

Mavis deposits \$1,000 today in a savings account that pays interest once a year. How much will Mavis have in three years' time if the interest rate is 12% p.a?

Solution

$$\begin{aligned} FV &= PV (1 + i)^n \\ &= 1000(1 + 0.12)^3 \\ &= 1404.93 \end{aligned}$$

Example 4

Freda deposits \$5,317 today in a savings account with an interest rate of 5% p.a. What is the value of Freda's deposit in four years' time?

Solution

$$PV = 5,317$$

$$i = 5\%$$

$$n = 4$$

$$FV = PV (1 + i)^n$$

$$\begin{aligned} FV &= 5,317 (1 + 0.05)^4 \\ &= \$6,462.85 \end{aligned}$$

Example 5

You own a bank term deposit that guarantees to pay you \$230,000 in six years' time. However, you are impatient. What amount of cash would you receive today if someone will buy the term deposit today?

- The appropriate discount rate is 20% p.a.

Solution

$$FV = 230,000$$

$$i = 20\%$$

$$n = 6$$

$$PV = FV (1 + i)^{-n}$$

$$= 230,000 (1 + 0.20)^{-6}$$

$$= \$77,026.53$$

Example 6

What is the compounding rate for each time period for a 18% nominal annual interest rate with monthly compounding?

Solution

The number of compounding periods each year is 12

$$\text{Interest rate per period} = 18\% \div 12 = 1.5\%$$