1 – ASSETS & INVESTMENTS

Investment → trade-off between current and future consumption
Return arises due to (1) time value of money, (2) inflation, (3) uncertainty (risk)

Asset allocation → distribution of an investor's wealth among different asset classes for investment purposes.

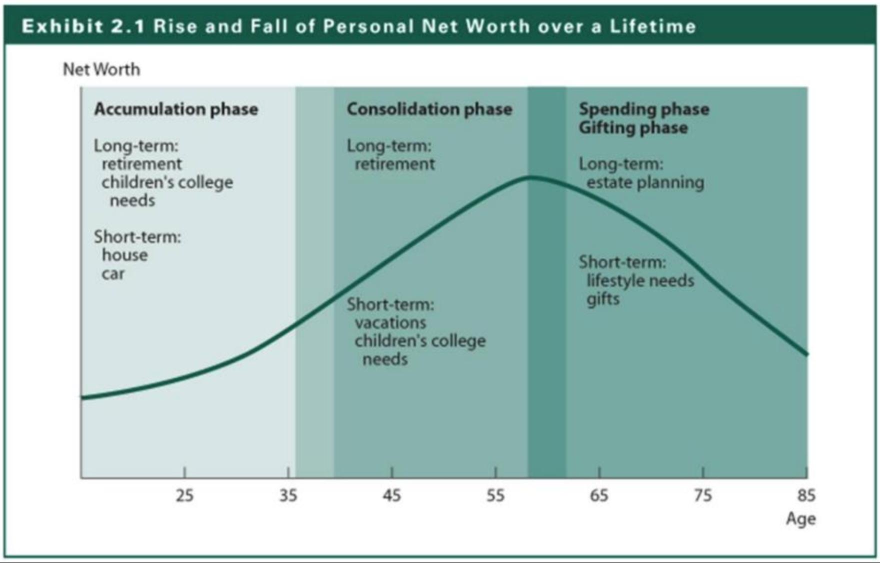
- Investor preferences (risk aversion, holding periods)

Asset class → group of securities that have similar characteristics.

- Examples: bonds, property, equity
- GROWTH assets: equity & property – high risk, high capital growth, moderate income
- DEFENSIVE assets: fixed income & cash – low/no risk, steady income

Investor life cycle

(1) Accumulation phase: early to middle years of working career.
(2) Consolidation phase: past midpoint of career, earnings > expenses
(3) Spending/Gifting phase: after retirement



Financial Planning

- Investor Policy Statement (IPS) is a summary document containing an investor's objectives, constraints and circumstances.

Objectives: **capital preservation** or **capital appreciation? Current income** → generate income, or **total return** → capital gains and reinvestment of current income? Also, risk objectives, **ability** to take risk (quantitative measurement) and **willingness** to take risk (qualitative assessment).
Required rate of return = minimum rate of return required on an investment

Constraints: liquidity, time horizon, taxes, regulation, unique circumstances

(1) Construct portfolio (allocate available funds to minimise risks and meet goals)
(2) Monitor & update (evaluate performance, modify accordingly)+

Performance measurements

(1) Holding Period Return (HPR)
HPR **(Price relative)** = Ending value / Beginning value
Holding Period Yield → HPY **(return)** = HPY = HPR – 1

(2) Arithmetic Mean Return
AM= Σ HPY / n
AM = sum of all annual HYPs / number of years

(3) Geometric Mean Return
GM= [π HPY] 1/n -1
GM = [product of all annual HPRs] $^{1/\text{number of years}}$ *– 1*

(4) Portfolio Returns
<u>Weighted average</u> of the returns on the securities comprising the portfolio.

Equal-weighted portfolio return:	$R = \frac{1}{N}\sum_{i=1}^{N} r_i$
Value [$]-weighted portfolio return:	$R = \sum_{i=1}^{N}\left(\frac{V_i}{V_p}\right) r_i$
Price-weighted portfolio return:	$R = \sum_{i=1}^{N}\left(\frac{P_i}{P_p}\right) r_i$

(5) Stock Market Indices

Expected return
E(r) = p1(r1) + p2(r2) + ... + pn(rn)
P_i = Probability for possible return; R_i = Possible return

Risk
Uncertainty of an investment.

(1) Variance

$$\sigma^2 = p_1.(r_1 - E(r))^2 + p_2.(r_2 - E(r))^2 + ... + p_n.(r_n - E(r))^2$$

(2) Standard deviation
σ

(3) Coefficient of variation

$$CV = \frac{\sigma}{E(r)}$$

Return distribution *(histogram plot of returns for an investment)*
Mean = average return
Median = value at 50%
Variance = how dispersed returns are from mean

Skewness = measure of symmetry
Kurtosis = relative number of observations that fall in the extreme ends of tails of distribution

Normal (Gaussian) distribution = symmetric with NO skewness or excess kurtosis

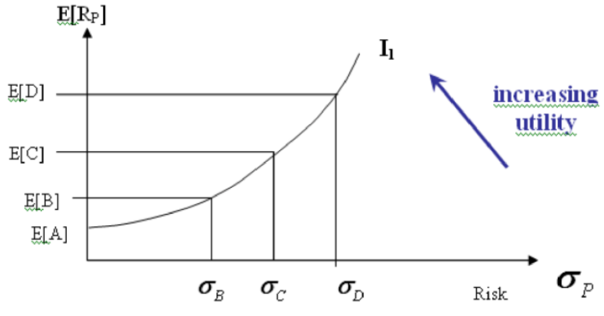
2 – PORTFOLIO THEORY

Assumptions concerning investors

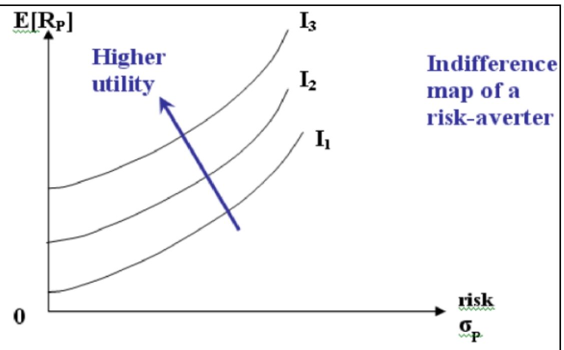
Risk aversion: investors minimizing risk
Rational: investors can consistently rank investments
Non-satiated: investors prefer more to less

Investor preferences & expected utility

Use expected return and S. D. to model investor preference.



- If distribution of expected returns is normal
- When utility functions are quadratic



- Investors can increase utility if they reach a higher indifference curve.

Portfolio return & risk

Portfolio return

$$E[R_P] = \sum_{i=1}^{n} w_i E[R_i] \qquad \sum_{i=1}^{n} w_i = 1$$

Portfolio risk

$$\sigma_P^2 = w_X^2 \sigma_X^2 + w_Y^2 \sigma_Y^2 + 2 w_X w_Y \rho_{XY} \sigma_X \sigma_Y$$

- Total risk: determined by average covariance between assets in portfolio.

$$\sigma_p^2 = \frac{\sigma_i^2}{n} + \left(1 - \frac{1}{n}\right)\sigma_{ij}$$

- Total portfolio risk decomposed into 2 components: Unsystematic Risk (diversifiable) & Systematic Risk (non-diversifiable)

Diversification

- A diversified portfolio contains securities moving in opposite directions > negative correlation → this reduces the **overall risk** of the portfolio, but not the expected return

- **Correlation coefficient (R_{AB}):** measure of correlation between 2 variables. R = 1 → perfect positive correlation; R = 0 → no correlation; R = -1 → perfect negative correlation.
- Complete risk reduction occurs in event of <u>perfect negative correlation</u> (R = -1)
- Minimum Variance Portfolio (MVP) → well-diversified portfolio of risky asset that hedge each other. Results in **lowest** possible risk for the rate of expected return:

$$x = \frac{\sigma_2^2 - \sigma_{1,2}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}}$$

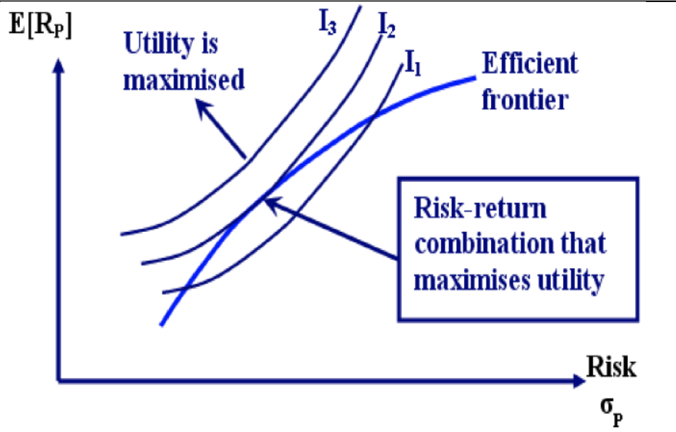
The Efficient Frontier

Risk averse want to decrease risk and increase E(R).

FIGURE 7.10

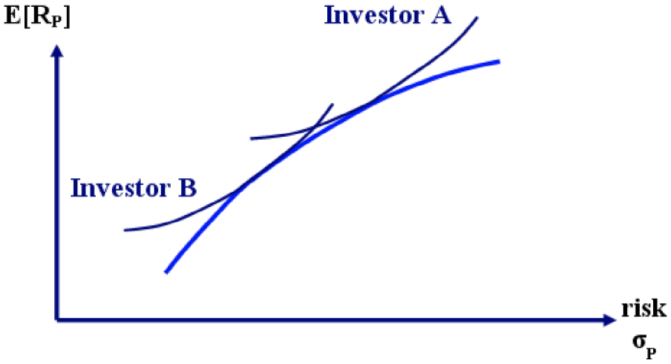
Investors will pick Portfolio 1 or 2 (not 3) as it has higher E(R) and lower risk.

Mean-variance Opportunity Set



Portfolios that have **least variance** for given expected return/**greatest expected return** for given variance.

<u>Optimal portfolio:</u> intersection of efficient frontier with indifference curve.

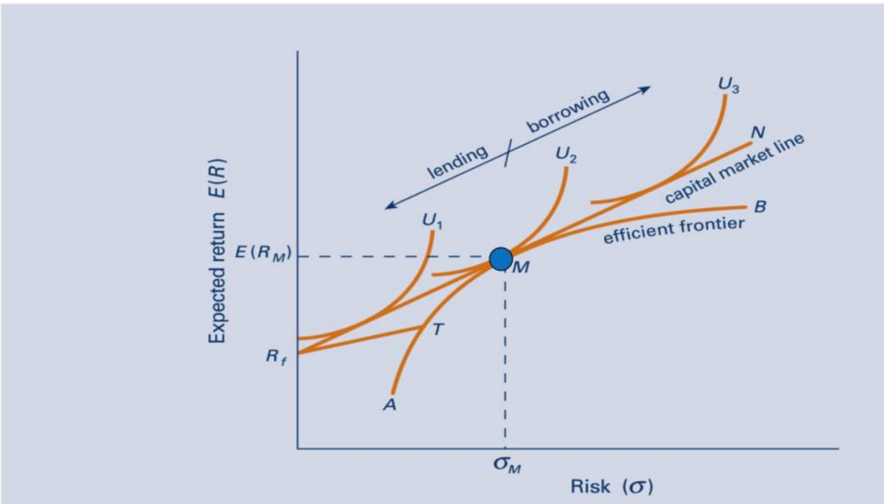


Investors have different risk preferences. Some (Investor A) may have greatest risk appetite than others (investor B).

Risk-free Assets *(R_f)* → provides risk-free rate of return (zero risk). Zero correlation with all other risky assets.

<u>Capital Market Line (CML)</u> → optimal portfolio with combination of risky and risk-free assets. It is the **intersection** between the efficient frontier and the original opportunity set. Once optimal risky portfolio is decided, choose between **borrowing/lending** the risk-free asset to maximize utility.

- Portfolio return according to CML: $$E(R_p) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M}\right)\sigma_p$$
- Investors can vary risk of portfolio investment by **changing weights** in R_f asset and market portfolio *M*
- Points to the left of tangency portfolio represents lending at risk-free rate (and right represents borrowing). Decision to borrow/lend based on <u>risk preferences</u> of the investor. Based on figure below:
  *Investor 1 (with U1) prefers to invest < 100% of wealth into Portfolio M and **lend** at risk-free rate.*
  Investor 2 (with U2) prefers to invest 100% of wealth in portfolio M.
  *Investor 3 (with U3) prefers to invest > 100% of wealth in Portfolio M and **borrow** at risk-free rate.*

 **(CML)**	Optimal: M Inefficient: R_fT Tangents with efficient frontier: R_fMN (represents different combinations of risk-free assets and portfolio M)

<u>3 – ASSET PRICING MODELS</u>

(1) Capital Market Line (CML) → measures optimal portfolio with combination of risky and risk-free assets.

(2) Capital Asset Pricing Model (CAPM) → determines E(R) on an individual asset. Studies the relationship between returns and systematic risk.

CAPM: $E(R_i) = R_f + \beta_i\left[E(R_M) - R_f\right]$