HYDRAULICS

Chpt 1

1. Def

- Fluid : A substance that deforms continuously under a shear force
  (in liquid or gas phase)
- shear stress applied to solid →
  → deform or bend to a certain level

liq	gas
not easily compressed	can be compressed
occupy fixed volume	fill entire volume

2. Unit

BG sys :

① length : foot (ft) ② time : s
③ mass : slug ④ force : pound (lb)
⑤ temperature : Fahrenheit (°F) or absolute temp in Rankine (°R)

$°R = °F + 459.67.$

$°C = (°F - 32)\,5/9$

EE sys :

① length : foot (ft) ② time : s
③ temp : abs temp in Rankine (°R)
④ mass : pound mass (lbm)
⑤ pound (lb)

Chpt 2 Fundmt Concepts

1. Continuum assumptions
- Valid if the mean free path of the molecules is (10^{-9}m for gases) much less than the smallest significant dimension of the problem ($< 10^{-6}$m)
  - both air & water satisfy under normal condition
- under continuum assumption:
  fluid prpts are continuous functions of time: e.g. $\rho = \rho(x, y, z, t)$

2. measures of fluid mass & weight
1) ave density: $\rho = m/\forall$
2) specific volume: $v = 1/\rho$
3) specific weight: $\gamma = \rho g$
4) specific gravity SG: non-dim

$$SG = \rho / \rho_{H_2O\ at\ 4°C}$$

At 0°C	
Substance	SG
water	1
mercury	13.6

fluid: ρ depends slightly on T & P under normal conditions.
gases: depend significantly on T, P

3. Ideal gas law:
- hypothetical gas consisting of identical particles of zero volume, with no intermolecular forces; elastic collisions with wall

law: $P = \rho R T$

P: abs pressure $P = p + p_0$

p: gauge pressure

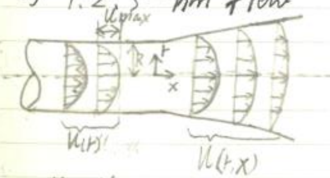
$T = °C + 273.15$ (K)

R: gas constant (air is $R = 286.9\ J/(kg \cdot k)$)
(depending on gas type & related to molecule weight)

4. Velocity field
1) velocity at a point: the instens velocity of the fluid particle passing through the point
2) $\vec{V} = \vec{V}(x, y, z, t)$ < with t: unsteady; without t: steady.

5. Types
1) dependence on time of prpts
① steady: $\frac{\partial \eta}{\partial t} = 0$ (η: flow prpt)
② un steady: $\frac{\partial \eta}{\partial t} \neq 0$
2) 1, 2, 3 - Dim flow

Uniform flow at cross section:
velocity const at cross-section



$u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$

3) velocity const or not
const $\vec{V} \Rightarrow$ uniform flow
non~ $\vec{V} \rightarrow$ non-~

Date No

6. Flow visualisation meathod

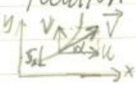
1) path lines

2) streakline:

- a line joining particles passing through a fixed location in space

line tangent to direction of velocity of every point in the flow at t

3) streamlines



S.L equation: $\boxed{\frac{dy}{dx} = \frac{v}{u}}$

$y = F(x)$ trace

flow particle hot pass streamline

4) timelines:

- a set of adjacent fluid particles that were marked at the instant in time.

Examp: velocity field given by $\vec{V} = 0.3x\hat{i} - 0.3y\hat{j}$

① velocity of a particle at (2, 8)

② equation of streamlines

③ plot streamlines passing through (2, 8)

④ if the particle passing through (x_0, y_0) is marked at $t = 0$, determine location of it at $t = 6$ s?

⑤ show the equation of the particle path is the same as equation of stream line

1) $\vec{V}_{(2,8)} = 0.6\hat{i} - 2.4\hat{j}$

2) $\frac{v}{u} = \frac{dy}{dx} = \frac{-0.3y}{0.3x} = -\frac{y}{x}$

$\therefore \frac{dy}{y} = -\frac{dx}{x}$

$xy = C$

3) $xy = 16$

infinite lines for $xy = C$ but only one passes (2, 8)

4) $u = \frac{dx}{dt} = 0.3x$

$v = \frac{dy}{dt} = -0.3y$

$x = x_0 e^{0.3t}$

$y = y_0 e^{-0.3t}$ $\Rightarrow t = 6$, $(x_0 e^{1.8}, y_0 e^{-1.8})$

7. Stress field

1) type of forces on fluid particles:

① body forces

② surface forces

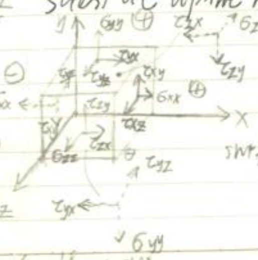
2) surface forces $\Rightarrow$ stresses

normal stress: $\sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n}$ (N/m^2)

~~normal~~ ~~shear~~: shear stress:

$\tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n}$

Stress at a point has nine components:

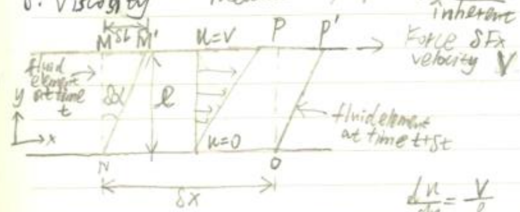


surface $\begin{cases} ⊕ \\ ⊖ \end{cases}$

for ⊖ surface positive direction point to negative axis

(internal friction.) Date No

8. Viscosity — measure of fluid's resistance to flow (inherent)



$\tau = \mu \frac{du}{dy}$

$W \sim \frac{du}{dy}$

$\tau \cdot A \cdot \frac{D}{2} \approx T$

$\frac{du}{dy} = \frac{V}{l}$

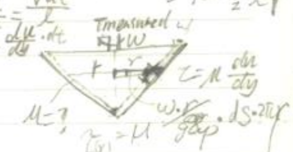
$\delta\alpha = \frac{\delta L}{l} = \frac{V dt}{l} = \frac{du}{dy} \cdot dt$

shear stress: $\tau_{yx} = \frac{\delta F_x}{\delta A_x}$

$\delta\alpha = \frac{\delta l}{\delta y} = \frac{\delta u \delta t}{\delta y} \Rightarrow \frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y}$

$\Rightarrow \frac{d\alpha}{dt} = \frac{du}{dy}$ ← rate of shear strain

rate of deformation



$\tau = \mu \frac{du}{dy}$

$\mu = ?$ $\tau_{(r)} = \mu \frac{\omega \cdot r}{gap} \cdot dS \cdot 2\pi r$

- shear stress is proportional to rate of deformation — Newtonian fluids

$\tau_{yx} \propto \frac{du}{dy}$

or: $\boxed{\tau_{yx} = \mu \frac{du}{dy}}$

where μ is dynamic viscosity

two measures of viscosity

① $\tau = \mu \frac{\partial u}{\partial z}$

② Kinematic viscosity ν: ②

$\nu = \mu / \rho$ $[\nu] = L^2 T^{-1}$ $\nu_{air} > \nu_{water}$ $[\mu] = ML^{-1}T^{-1}$

$\mu_{water} > \mu_{air}$

- Fluid viscosity depends on fluid type and temp

— usually assume fluid Newtonian

i.e. $\mu = const \neq f(\tau)$

(Not for all cases)

6

Q1-2:
exp following using F-L-T sys
a) P b) Pressure
c) power d) energy
e) mass flux
f) Volume flow rate

a) $P = FT^2/L^4$
b) $P_{ress} = F/L^2$
c) $P_{ower} = FL/T$
d) $energy = FL$
e) $\dot{m}_{flux} = FT^2/L^3$
f) $V_{flow\,rate} = L^3/T$

9. type of fluids
1) Newtonian fluids
- shear stress is proportional to deformation rate: $\tau = \mu \frac{du}{dy}$
- most of common fluids: e.g. water, air, gasoline

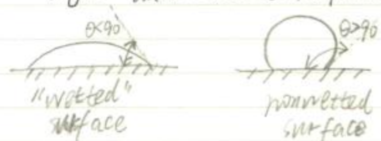
2) Non-Newtonian fluids
- shear stress not proportional to deformation rate
$\tau_{yx} = k\left(\frac{du}{dy}\right)^n$
- bingham fluids, shear thinning and shear thickening fluids ...

10. Surface tension
1) develops ~~at~~ interfaces with other liquids or solids due to the attraction between molecules
2) surface tension: force/length (N/m)
3) interface acts like a stretched elastic membrane (with a contact angle θ) creating surface tension σ
4) θ and σ depend on the type of liquid and contact surface

at 20°C. ($1 mN/m = 10^{-3} N/m$)

Liquid	surface tension σ (mN/m)	contact angle θ
water	72.8	~0
ethyl alcohol	23	

Exmp of Non-Newtonian fluid: - OOBleck = cornstarch + water (shear thickening fluid: as τ↑, μ↑)
- painting is the opposite (shear-thinning fluid: as τ↑, μ↓)



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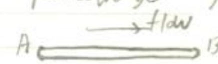
10.1 pressure inside a drop of fluid
σ: surface tension (N/m)
$\Delta p = p_i - p_e$
(p_i: internal pressure, p_e: external pressure)
force around the edge: $2\pi R\sigma$
force due to pressure difference: $\Delta p \pi R^2$
balance: $2\pi R\sigma = \Delta p \pi R^2$
$\Rightarrow \Delta p = \frac{2\sigma}{R}$

10.2 Eng application
- insignificant for large scale of motions
- important for some small scale motions:
e.g. capillary rise (or depression)

balance: surface tension = weight
$\hookrightarrow 2\pi R\sigma\cos\theta = \gamma\pi R^2 h$
$\hookrightarrow h = \frac{2\sigma\cos\theta}{\gamma R}$, $h \propto \frac{1}{R}$

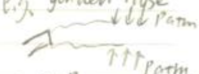
11. Compressibility of fluids
1) incompressible fluids
- density change negligible: $\frac{D\rho}{Dt} = 0$
- liquids & gases with small Mach number ($M = V/c < 0.3$, c: speed of sound, V: flow velocity)

* pressure in flow
a) Law of hydrostatic pressure (no motion)
(still applies if fluid in motion only in horizontal)
b) pipe flow
(typically, horizontal change >> vertical pressure change)



neglect vertical variation in pressure + assume uniform across the cross-section

b) Free jets
(fluid jets open to atm all sides)
e.g. garden hose



- $p = p_{atm}$ everywhere in free jet

2) compressible fluids
- density change not negligible
- e.g. flow of gases ($M > 0.3$); some flow of liquids under extremely high pressure ~~(water~~ (water hammer)

12. Bulk modulus

Bulk compressibility modulus (or modulus of elasticity)
- ratio of pressure change to relative density change (compressibility)
- unit: N/m^2, Pascal
- $E_v = \frac{dp}{(d\rho/\rho)}$

13. compression & expansion of gases
- when gases compressed (expanded), relationship between pressure and density depends on the nature of the process

1) under constant temperature condition ($T-$):

$$\frac{p}{\rho} = \text{Const}, \quad E_v = p$$

2) under isentropic conditions (frictionless compression and no heat exchange with surroundings)

$$\frac{p}{\rho^k} = \text{Const}, \quad E_v = kp$$

k: ratio of the specific heat at const pressure, c_p, to the specific heat at const volume c_v ($R = c_p - c_v$)

(Note: pressures are abs pressures)

14. speed of sound,
1) disturbances in fluids travel at speed of sound.
2) speed of sound in fluids:

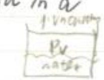
$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{E_v}{\rho}}$$

3) for gases undergoing isentropic process $E_v = kp$

$$c = \sqrt{kp/\rho}$$

ideal gas: $c = \sqrt{kRT}$

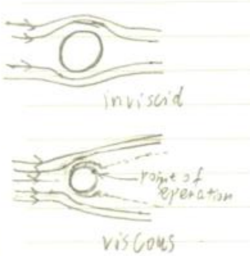
15. Vapour pressure
- a pressure developed in a vacuum space left above the liquid in a closed container



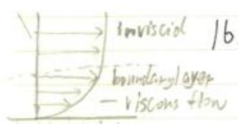
- $T\uparrow$, $p_v\uparrow$
- boiling: when the absolute pressure reaches p_v
- importance in engineering appli:
  for fluid passing valve and pumps,
  when $p < p_v \Rightarrow$ vapor bubble (low p)
  $\Rightarrow$ swept along high $p \Rightarrow$ bubble collapse
  $\Rightarrow$ may damage the structures
  — cavitation

16. Classification of fluid motions

16.1 viscous and inviscid flows



- all real fluids are viscous
- flows assumed inviscid when viscosity not important
  - total drag on an airfoil
- analysis of inviscid flows much simpler than viscous
  - laplace equation (for irrotational flows)
  - Navier-stokes equations (for viscous flows)



16.2 laminar & turbulent flows

1) laminar flow
   - fluid particles move in smooth layers
2) turbulent flow
   - fluid particles mix rapidly with random three-dimensional velocity fluctuations.

① $\vec{V} = u\hat{i}$ → laminar

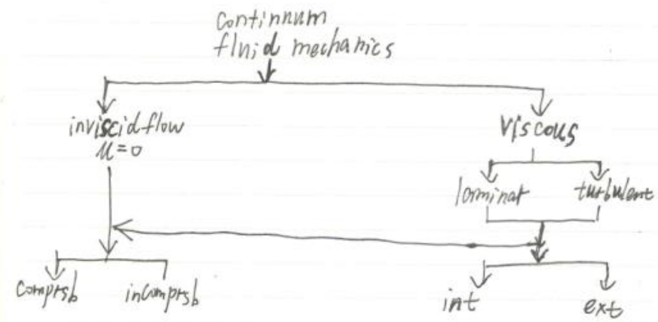
② $\vec{V} = (\bar{u}+u')\hat{i} + v'\hat{j} + w'\hat{k}$ → turbulent.

16.3 Internal & external flows

1) internal flows
   - flows completely bounded by solid surfaces. (e.g. flow in pipes)
2) external flows
   - flows over bodies immersed in unbounded fluid (e.g. flow around a submarine)

- both int & ext flows maybe laminar/turbulent compressible / incompressible

~~classification~~ Classification of fluid motion

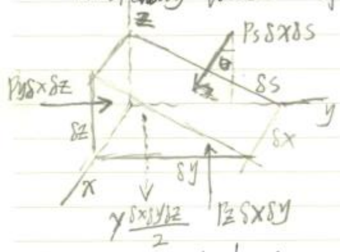


chpt3 Hydrostatics

12.

Fluid statics $\begin{cases} \text{no relative motion between fluid particles } (\frac{du}{dy}=0) \\ \text{no shear stress } (\tau = \mu\, du/dy = 0) \\ \text{normal stress (pressure) exists} \end{cases}$

1. Pascal's law
   arbitrary wedge-shaped element:



Newton's 2nd law

$\Sigma F_y = m a_y$; $p_y \delta x \delta z - p_s \delta x \delta s \sin\theta = \rho \frac{\delta x \delta y \delta z}{2} a_y$

$\Sigma_z = m a_z$; $p_z \delta x \delta y - \gamma \frac{\delta x \delta y \delta z}{2} - p_s \delta x \delta s \cos\theta = \rho \frac{\delta x \delta y \delta z}{2} a_z$

Since $\delta y = \delta s \cdot \cos\theta$; $\delta z = \delta s \cdot \sin\theta$

$\Rightarrow p_y - p_s = \rho a_y \frac{\delta y}{2}$

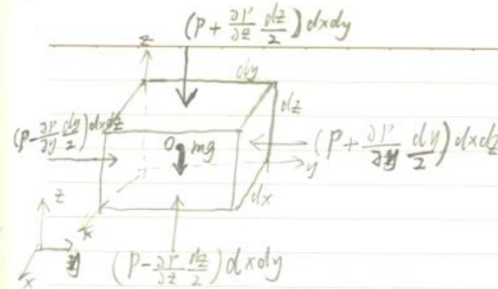
$p_z - p_s = (\rho a_z + \gamma)\frac{\delta z}{2}$

when $\delta y \to 0$ and $\delta z \to 0 \Rightarrow p_s = p_y = p_z$

$\Rightarrow$ pressure at a point is indpdnt of dirctn (Pascal's law) – a scalar prpty of fluid.

2. Pressure variation in a static fluid.
   1) body force 2) surface forces.

13



Total forces:

X: $\Sigma F_x = -\frac{\partial p}{\partial x} dx\, dy\, dz$

Y: $\Sigma F_y = -\frac{\partial p}{\partial y} dx\, dy\, dz$

Z: $\Sigma F_z = -\frac{\partial p}{\partial z} dy\, dx\, dz - mg$

Applying Newton's 2nd law

$\left.\begin{aligned} \Sigma F_x &= m a_x \\ \Sigma F_y &= m a_y \\ \Sigma F_z &= m a_z \end{aligned}\right\}$ since $m = \rho\, dx\, dy\, dz$

$\partial p/\partial x = -\rho a_x$

$\partial p/\partial y = -\rho a_y$

$\partial p/\partial z = -\rho (g + a_z)$

$\because a_x = a_y = a_z = 0$

$\begin{cases} \partial p/\partial x = 0 \\ \partial p/\partial y = 0 \\ \partial p/\partial z = -\rho g = -\gamma \end{cases}$

Restrictions:
1) static fluid
2) gravity as only body force
3) z-axis vertical and upward.

$\because$ p only depends on z

$\Rightarrow \frac{dp}{dz} = -\rho g = -\gamma$

2.1 Pressure change – Integral form:
for incompressible fluids, integrate $\frac{dp}{dz} = -\rho g = -\gamma$

$\int dp = -\gamma \int dz \Rightarrow p = -\gamma z + const$

$\Rightarrow p + \gamma z = Const$

or $\underbrace{\frac{p}{\gamma} + z}_{\text{piezometric head}} = Const$

at two points 1, 2 in liquid:

$$\boxed{\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2} \quad \text{or} \quad p_2 - p_1 = \gamma\underbrace{(z_1 - z_2)}_{\text{pressure head (m)}} = \Delta p$$

Exmp: air / lqd, free surface, $p_1 = 0$ (gage), $z_1 = 0$, $z_2 = -h$

At $z_1 = 0$, $p_1 = 0 \rightarrow \text{Const} = 0$

$p_2/\gamma - h = 0 \Rightarrow p_2 = \gamma h = \rho g h$ (gage)

2.2 Absolute pressure & ~~gause~~ gage pressure

$p_{abs} = p_{gage} + p_{atm}$

— pressure values must be stated with respect to a reference

2.3 pressure transmission throughout a stationary fluid

— pressure is a const on a line if
- line horizontal
- stationary fluid is uniform and
- stationary fluid is continuous.

3. Measurement of pressure

o pressure measurement devices:
— mercury barometer: for atmospheric pressure.
— manometers: for gage pressure using liquid columns in vertical / inclined tubes.
— mechanical & ~~electrical~~ electronic pressure measuring devices.

o in engineering, normally use gauge pressure. for abs pressure, p_{abs} must be indicated.

3.1 Mercury Barometer — find p_{atm}

$p_{atm} = \rho_{merc} g h$

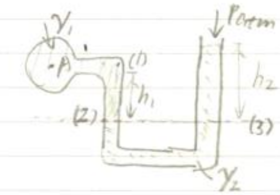
$p_{gage} = p_{abs} - p_{atm}$

3.2 Piezometer tube — to find p_A or p_1

$p_1 = p_{atm} + \gamma_1 h_1$ (abs)

$p_1 = \gamma_1 h_1$ (gage)

3.3 U-Tube manometer — p_A



$p_2 = p_3$, $p_A = p_1$

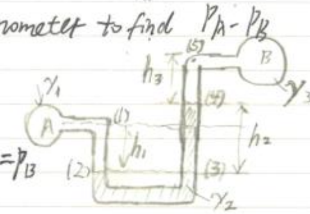
$p_2 = p_1 + \gamma_1 h_1 = p_A + \gamma_1 h_1$

$p_3 = p_{atm} + \gamma_2 h_2$

$p_A = p_{atm} + \gamma_2 h_2 - \gamma_1 h_1$ (abs)

$p_A = \gamma_2 h_2 - \gamma_1 h_1$ (gage)

3.4 Differential U-tube manometer to find $p_A - p_B$



$p_A = p_1$ $\quad p_1 + \gamma_1 h_1 = p_2$

$p_2 = p_3$ $\quad p_3 - \gamma_2 h_2 = p_4$

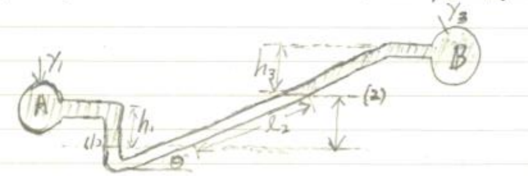
$p_4 - \gamma_3 h_3 = p_B$

Adding up: $p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$

Differential pressure:

$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$

Inclined-tube manometer to find $p_A - p_B$



inclined for better reading accuracy

$p_A + \gamma_1 h_1 - \gamma_2 l_2 \sin\theta - \gamma_3 h_3 = p_B$

OR: $p_A - p_B = \gamma_2 l_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$

4. ~~Hydrau~~ Hydrostatic force on submersed surfaces

- quantities of interests
  - ~~magn~~ magnitude of force
  - direction of force
  - location of action of force
- basic equation:

$$P = P_0 + \gamma h \quad \text{or} \quad P = \gamma h \quad \text{with} \quad \gamma = \rho g$$

- method — integration

4.1 Hydrostatic force on a flat surface

- magnitude: $F = P_c \cdot A$, P_c is pressure at centroid
- direction: perpendicular to surface
- location of action or centre of pressure — to find.

$F = P\,dA$

$F_R = \int_A P\,dA = \int_A \rho g h\,dA = \cancel{\int_A \rho g h} \int_A \rho g y \sin\theta\,dA$

$\quad = \gamma \sin\theta \int_A y\,dA$

$\int_A y\,dA = y_c A \rightarrow$ first moment of area

~~(y_c: centroid of~~

(y_c: coordinate of centroid of area A)

$F_R = \gamma A y_c \sin\theta = \gamma A h_c = P_c A$ (Force magnitude)

(P_c: gage pressure at centroid of A)

F_{total} = pressure at centroid × total area.

Location of resultant force — pressure centre

moment of resultant ~~force~~ a force about x axis

= moment due to distributed force about the same axis

$y' F_R = \int_A y P\,dA = \int_A \gamma y^2 \sin\theta\,dA$

$$y' = \frac{\gamma \sin\theta \int y^2 dA}{F_R} = \frac{\gamma \sin\theta \int_A y^2 dA}{\gamma \sin\theta\, y_c A} = \frac{\int_A y^2 dA}{y_c \cdot A} \quad ①$$

$\int_A y^2 dA = I_x \longleftarrow$ second moment of the area about x-axis

with parallel axis theorem:

$$I_x = I_{xc} + A y_c^2 \quad ②$$

(I_{xc}: second moment of the area with respect to an ~~area~~ axis passing through its centroid and parallel to x-axis)

$① + ② \Rightarrow y' = \frac{I_{xc}}{A y_c} + y_c$ (Force acting location)

↳ resultant does Not pass through centroid as $I_{xc}/(y_c A) > 0$

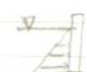
similarly, $x' = \frac{I_{xy}}{y_c A} = x_c + \frac{I_{xyc}}{y_c A}$

where $I_{xy} = \int_A xy\,dA = I_{xyc} + A x_c y_c$ is product of inertia with respect to x & y

— Direction of resultant force: perpendicular to the plane.

Exmps:

A: vertical wall, width b, high h



Force on plate

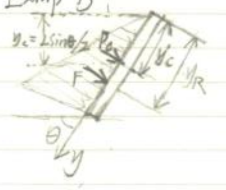
$F_R = P_c A = \gamma y_c A = (\gamma h/2)(bh) = \gamma b h^2/2$

$P_c = \gamma h/2$ pressure at centroid.

pressure center: $y_R = y_c + \frac{I_{xc}}{A y_c}$, with $I_{xc} = \frac{1}{12} b h^3$, $A = bh$ $\Rightarrow y_R = \frac{2h}{3}$

Date No

Exmp B:



Force on the plate: $F_R = P_c A$

centroid $y_c = (L\sin\theta)/2$

Pressure at centroid

$P_c = \gamma y_c = \gamma(L\sin\theta)/2$

Area: $A = bL$

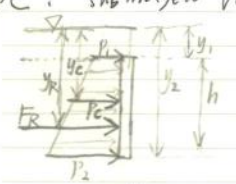
$\Rightarrow F_R = \gamma(L\sin\theta)/2 \times bL = (\gamma b L^2 \sin\theta)/2$

Pressure center $y_R = y_c' + \frac{I_{xc}}{A y_c'}$ with $y_c' = L/2$

with $I_{xc} = \frac{1}{12} bL^3$, $A = bL$

$\Rightarrow$ Pressure center $y_R' = 2L/3$

exmp C: submerged vertical wall, width b, height h



Force on plate: $F_R = P_c A$

centroid: $y_c = \frac{(y_1+y_2)}{2}$

pressure at centroid:

$P_c = \gamma y_c = \gamma(y_1+y_2)/2$

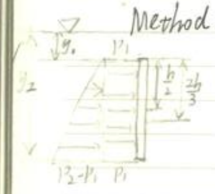
Area: $A = bh$

Force on the plate: $F_R = bh\gamma(y_1+y_2)/2$

Location of action (Method 1):

$y_R = y_c + \frac{I_{xc}}{A y_c}$ with $I_{xc} = \frac{bh^3}{12}$, $A = bh$

$y_R = \frac{(y_1+y_2)}{2} + \frac{\frac{bh^3}{12}}{bh} \times \frac{2}{y_1+y_2} = \frac{(y_1+y_2)}{2} + \frac{h^2}{6(y_1+y_2)}$



Method 2 for y_R:

decompose pressure prism into two;

part 1: rectangular, P_1

part 2: triangular, $P_2 - P_1$

Pressure centers:

part 1: $y_{R_1} = y_1 + h/2$

Part 2: $y_{R_2} = y_1 + 2h/3$

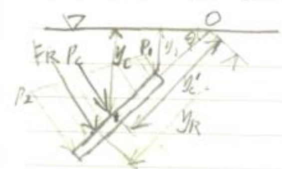
Date No

Using moment balance

$F_R y_R = F_1 y_{R_1} + F_2 y_{R_2}$

where $F_1 = \gamma y_1 hb$, $F_2 = \gamma h^2 b/2$ $\Rightarrow y_R$

exmp D: submerged by y_1 inclined wall, width b, length L



Force on plate: $F_R = P_c A$

Centroid: $y_c = y_1 + (L\sin\theta)/2$

pressure at centroid:

$P_c = \gamma y_c = \gamma[y_1 + (L\sin\theta)/2]$

Area: $A = bL$

Force on the plate: $F_R = P_c A = bL\gamma[y_1 + (L\sin\theta)/2]$

Pressure center (method 1):

$y_R = y_c' + \frac{I_{xc}}{A y_c'}$, with $I_{xc} = \frac{1}{12} bL^3$ and $A = bL$

$y_R = \frac{6y_1^2 + 6Ly_1\sin\theta + 2L^2\sin^2\theta}{3\sin\theta(2y_1 + L\sin\theta)}$

4.2 Force on a curved surface

- decompose force into Fv (vertical) and FH (horizontal)
- FH: Force on projected vertical plane surface
- Fv: Weight of liquid directly above.
  (Real: ↓ Imaginary: 浮↑)

  Fv passes ~~the~~ through the center of ~~the~~ gravity of the liquid volume directly above the curved surface.
- Resultant force: $F_R = \sqrt{(F_H)^2 + (F_V)^2}$