Financial Mathematics

Note: always draw timeline, as it makes it clearer

The aim of financial mathematics is to reduce a series of cash flows (asset) to a common monetary base, taking into account the time value of money for deciding:

- Which assets are more valuable
- Appropriate prices to pay for assets

Simple and Compound Interest

- Simple interest – the amount of interest paid per period doesn't vary and is based on the initial cash flow (aka principle or present value (PV))
  - It is used extensively in the money market
  - Interest = Present Value x r
    - Where r = aggregate interest for the entire period
  - Future Value = Present Value + Interest = Present Value + (Present Value x r)
  - $\therefore$ *Future Value (FV) = PV (1 + r)*
- Compound Interest – interest is received during each set period (compounding period) and interest is earned on the principle plus the interest (interest earned on interest)
  - $FV = PV (1 + r)^t$
    - Where r = interest rate per period
    - Where t = number of periods

Effective Interest Rate

- It is the actual interest received/paid for a stated nominal interest rate
- It can be used to convert a compound interest rate into a simple interest rate OR to convert one type of compound interest rate (e.g. annually) into another type of compound interest rate (e.g. semi-annually)
- *Effective Rate (ER)* $= (1 + \frac{nominal\ rate}{m})^{mn} - 1$
  - Where m = how often it is compounded (compounding period)
  - Where n = period you are trying to calculate the effective rate for (time in years)
- Note: credit card interest is compounded daily
- If the compounding rate is not compounded annually (but the question it is talking about something in 'x' years) must use the effective rate
  - E.g. if the current interest rate is 7% compounded quarterly, how much money would Shaun need to put aside today to buy the new iPhone in 3 years' time?
    - $ER = (1 + \frac{0.07}{4})^{4 \times 1} - 1 = 0.071859$
    - $\therefore PV = \frac{2999}{(1+0.071859)^3} = \$2435.36$

Continuous Compounding

- This is a theoretical case where compounding is assumed to occur at every instant in time
- $FV = PV e^{rt}$
  - Where r = continuously compounded rate of return
  - Where t = time period of investment/borrowing
  - Where e = 2.718282

Present Value of a Single Amount

- It is the present dollar worth of a payment to be received in future taking into account the time value of money
    - PV is also its market price
- FV = PV $(1 + r)^t$ → $\therefore PV = \frac{FV}{(1+r)^t}$
    - Where r = interest rate for the entire period
        - r is aka discount rate, OC, cost of capital, required rate of return
        - $\frac{1}{(1+r)^t}$ = discount factor (measures the present value of one dollar received in year t)
        - Note: discounting is the process of taking a future cash flow and seeing what it is worth today
        - OC – best return you can get on something else of similar risk
            - When you discount the expected cash flows by the opportunity cost of capital, you are asking how much investors in the financial markets are prepared to pay for a security that produces a similar stream of future cash flows
            - OC is a concept about how discount rates are determined
            - Opportunity cost of capital: the rate of return that your company's shareholders could get by investing on their own at the same level of risk as the proposed project
                - The OC of capital depends on the risk of the cash flows to be valued
                - It is a standard measure of profitability that we use to calculate how much the project is worth
- The longer you have to wait for your money, the lower its present value is
- ↑ compound = ↑ FV and ↓ PV