Lecture 1 Art gallery problem

Sunday, 4 March 2018
10:25 pm

General TSP	No restrictions on the cost function.
Metric TSP	All edge cost are symmetric and fulfill the triangle inequality: $c(u, v) \leq c(u, w) + c(w, v), \forall u, v, w \in V$
Euclidean TSP	The vertices correspond to points in a d-dimensional space, and the cost function is the Euclidean distance.

APX Hard	Cannot approximate the answer within constant factor

Art gallery problem
What is the smallest number of guards needed to guard any polygon with n vertices?

- X can see Y if the segment is completely within the polygon

Definition of a convex set:
Take any two points in the polygon. A segment joining these two points must be entirely within the polygon.

Every simple polygon can be triangulated

Lemma: Every simple polygon with >3 vertices has a diagonal

Proof

- Find the lowest vertex by sweep lining up
- A and B are adjacent vertices
- Either the rest of the shape is above the AB diagonal (1), or intersects the AB diagonal (2)
    - If (1) then there exists a diagonal
    - If (2) then there must be a vertex Q which can be connected to V
- QED

Does a triangulation always exist?

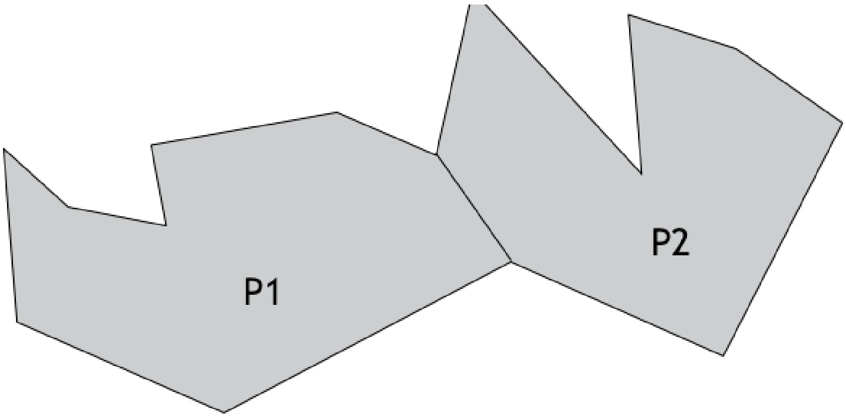
- Use induction, base case n=3

The triangulation of a simple polygon can be 3-coloured

Lemma: Every triangulation consists of n-2 triangles

Proof

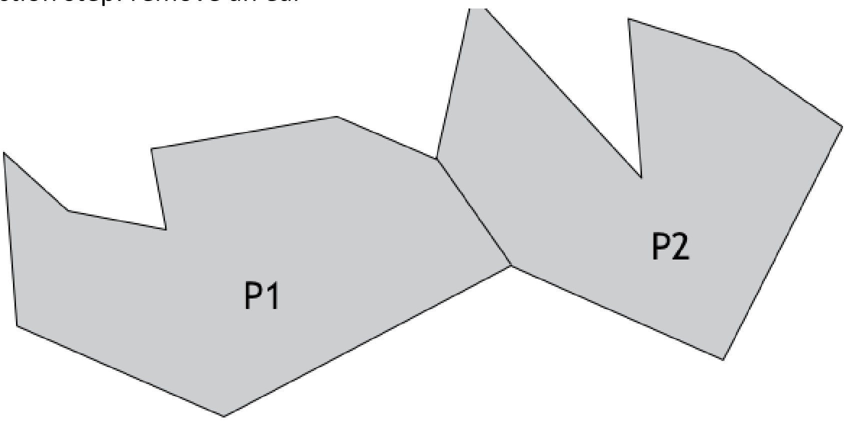
- Use induction, base case n=3
- Let P1 have m1 vertices
- Let P2 have m2 vertices
    - m1 + m2 == n+2 because we count the diagonal's vertices twice



Assign a colour to each vertex so that no two adjacent vertices have the same colour.
Does a 3-colouring always exist?

Dual graph	The dual graph of a plane graph G is a graph that has a vertex for each face of G. The dual graph has an edge whenever two faces of G are separated from each other by an edge.

Proof

- Consider the dual graph
- The maximum degree of any node is 3 (triangles have 3 sides where which a another triangle can form)
- The dual graph has at least 2 ears (for n>3)
- Then prove by induction
  - Base case n=3
  - Induction step: remove an ear
  - 

RESULTS:

1. Every simple polygon can be triangulated.
2. The triangulation of a simple polygon can be 3-coloured.
3. Every simple polygon with n vertices can be guarded with $\lfloor n/3 \rfloor$ guards.

Why floor (n/3)?