

Business Finance

Lecture 1

Investment analysis - The point of view of an investor, how much should I pay for a certain share or bond in regards to its potentiality of profit? Thus, valuation of stocks, bonds and derivatives.

Corporate finance - The point of view of a manager or specifically the CFO. This includes:

- Capital Budgeting - what investments to make
- Capital structure - how to finance these investments, debt or equity?
- Dividend policy - what to payout to shareholders

Finance function

-The main goal of managers is to maximise the market value of the firm - this maximises the wealth of shareholders as the firm is valued higher.

- Value of firm = present slue of future expected cash flows
- Shareholder wealth = present value of shareholder's future expected cash flows.

Market value of a firm

$$Firm\ Value = \sum_{t=1}^n \frac{E(CF_t)}{(1+r)^t}$$

$E(CF_t)$ = expected cash flows received at the end of period t

n = number of periods over which cash flows are received

r = rate of return required by investors.

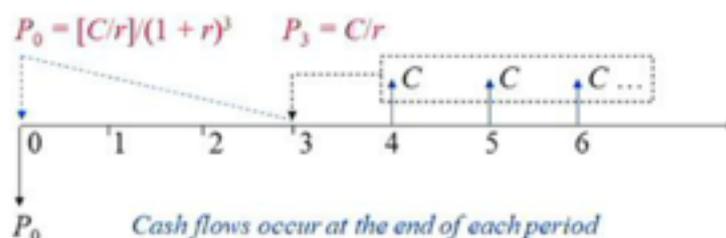
Lecture 2

Financial mathematics

Perpetuity - an equal periodic cash flow that goes on forever. E.g - evaluating a piece of land, land never expires and goes on forever.

- ❖ The present value of a perpetuity is...
 - ❖ $P_0 = C/(1+r) + C/(1+r)^2 + \dots + C/(1+r)^n + C/(1+r)^{n+1} + \dots$
 - ❖ $P_0 = C [1/(1+r) + 1/(1+r)^2 + \dots + 1/(1+r)^n + 1/(1+r)^{n+1} + \dots]$
- ❖ As n approaches ∞ , $[1/(1+r) + 1/(1+r)^2 + \dots + 1/(1+r)^n + \dots]$ approaches $1/r$
- ❖ So, the present value of a perpetuity, $P_0 = C/r$

-Deferred perpetuity is an equal, periodic cash flow that starts at some future date and then goes on forever. E.g a starting company that doesn't pay cash flows in the first few periods but then later in the future for example at period 4.



E.g - a prize that guarantees you \$10,000 per year forever with the first payment to be made at the end of year 1. How much is the prize worth if the interest rate were 10% p.a?

-Perpetuity - $P_0 = 10,000/0.10 =$ present value is \$100,000

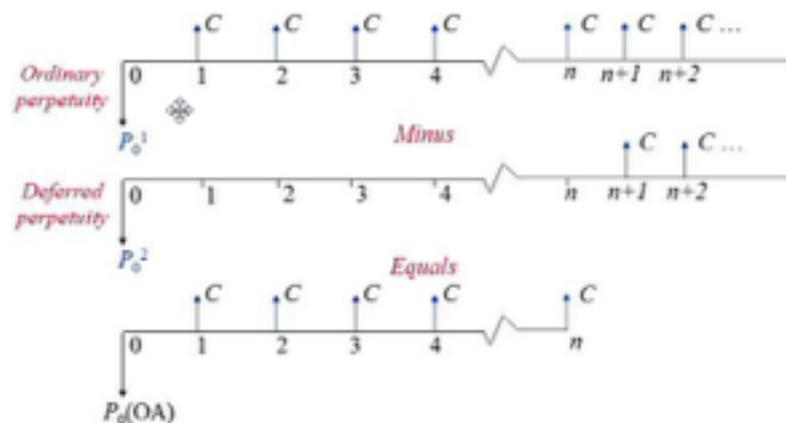
-What if the prize's value was deferred to the end of year 4, the present value or discounted value at year 3 is still \$100,000, then we take this value and discount it 3 more times.

$$\diamond P_3 = 10000/0.10 = \$100,000 \dots$$

$$\diamond P_0 = 100000/(1.10)^3 = \$75,131$$

Ordinary annuities - series of equal periodic cash flows occurring at the end of each period and lasting for n periods - e.g mortgage, loans, bonds. There is a maturity date for these payments.

-An n period annuity can be valued as the difference between two perpetuities, e.g an ordinary perpetuity MINUS a deferred perpetuity (e.g first payment at the end of period 4), thus we get the annuity of the payments fro period 0 to 4.



$$\diamond P_0(OA) = P_0^1 - P_0^2$$

$$\diamond P_0(OA) = C/r - [C/r][1/(1+r)^n]$$

\diamond Simplifying the above equation, we get...

$$\diamond P_0(OA) = [C/r][1 - 1/(1+r)^n] \quad \text{or}$$

$$\diamond P_0(OA) = [C/r][1 - (1+r)^{-n}]$$

Future value of annuities

-the future value of an annuity can then be obtained by computing the future value of the above present value at time period n.

Examples:

-Suppose you invest \$1,000 every year for 10 years with an annual return of 10%

-a) Future values at the end of 10 years

$$\diamond \text{In 10 years: } F_{10} = [1000/0.10][1.10^{10} - 1] = \$15,937$$

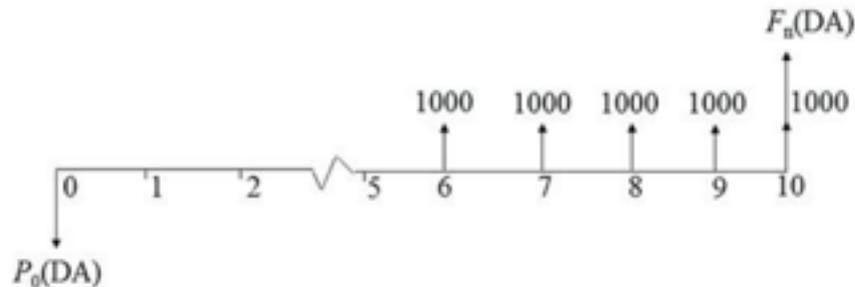
b)Present values

$$\diamond \text{Over 10 years: } P_0 = [1000/0.10][1 - 1.10^{-10}] = \$6,145$$

-Deferred ordinary annuities

-Series of the first equal, periodic cash flows occurs at a future date.

-E.g - plan to invest \$1,000 every year for five years but only want to start at the end of year 6, at 10% p.a. No funds are invested during years 1-5.



-Future value - (same as normal annuity as the same amount of cash flows in the future)

-Present value however is different as we must then again discount again by 5 periods.

❖ Present value of deferred annuity

$$❖ P_5 = [1000/0.10][1 - 1.10^{-5}] = \$3,791...$$

$$❖ P_0 = 3791/(1 + 0.10)^5 = \$2,354$$

-Annuity due - series of equal periodic cash flows occurring at the BEGINNING of each period, e.g rent is paid at the beginning of the month.

-So e.g the first cash flow is at the beginning of period 1, which then essentially as the same as the end of period 0. So then we could assume that the first payment occurred in the back.

-Thus, the present value of an annuity due is equivalent to the present value of an ordinary annuity compounded one additional period.

$$❖ P_0(AD) = [C/r][(1 - (1 + r)^{-n})[1 + r]]$$

-The future value of an annuity due is equivalent to compounding by one additional period the future value of an ordinary annuity.

$$❖ F_n(AD) = [C/r][(1 + r)^n - 1][1 + r]$$

E.g - you invest \$1,000 every year but at the beginning of each year, whats the future and present values of the investment earning at 10% return p.a over 10 years and 50 years?

❖ Future values at the end of 10 and 50 years are...

$$❖ \text{In 10 years: } F_{10} = [1000/0.10][1.10^{10} - 1][1.10] = \$17,530$$

$$❖ \text{In 50 years: } F_{50} = [1000/0.10][1.10^{50} - 1][1.10] = \$1,280,300$$

❖ Present values of the investments are...

$$❖ \text{Over 10 years: } P_0 = [1000/0.10][1 - 1.10^{-10}][1.10] = \$6,759$$

$$❖ \text{Over 50 years: } P_0 = [1000/0.10][1 - 1.10^{-50}][1.10] = \$10,906$$

Effective interest rate

- interest is typically not earned or paid on an annual basis, even though rates are quoted on an annual basis, the compounding interval could be monthly however and thus the effective annual interest rate is different to the nominal rate.

$$r_e = (1 + r/m)^m - 1$$

where r/m is the *per period* interest rate

- The effective rate will always be higher than the nominal rate if the interval is compounded.
- E.g rate of 8% is compounded semi annually - $(1+0.08/2)^2-1 = 8.16\%$

❖ Future value of P_0 today

$$F_n = P_0 \times (1 + r)^n$$

❖ Present value of F_n at time n

$$P_0 = F_n / (1 + r)^n = F_n (1 + r)^{-n}$$

❖ Present value of a perpetuity

$$C/r$$

❖ Present value of a deferred perpetuity

$$[C/r] / (1 + r)^n$$

❖ Present value of ordinary annuity

$$P_0(OA) = [C/r][1 - 1/(1 + r)^n]$$

❖ Future value of ordinary annuity

$$F_n(OA) = [C/r][(1 + r)^n - 1]$$

Lecture 3

Valuation of debt securities

-Short term debt instruments - mature within the year usual in 90/180 days, issuer has contractual obligations e.g treasury bills, bank bills etc. Face value is the amount paid at maturity, the interest is earned by investors based on the difference between price paid for the debt instrument and the face value that is paid back.

-Long term debt instruments - mature longer than one year.

-Coupon rate - the interest rate promised by the issuer expressed as a percentage of the face value, e.g 10% coupon rate is 10% times the face value.

The valuation principles

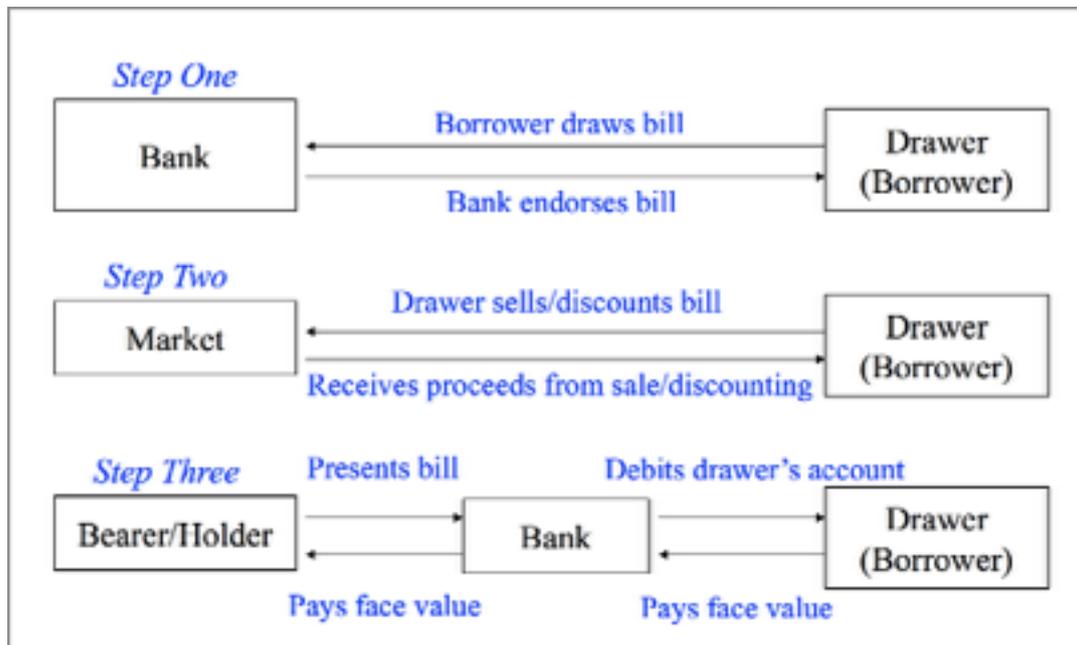
-the price of a security today is the present value of all future expected cash flows discounted by the required rate of return

-The valuation variables are: market price, future expected cash flows and yield to maturity or required rate of return.

Valuing discount securities

-There is no interest payment within discount securities, the interest is the difference between the face and present value of the security.

-With bank accepted bills, some firms that may not have such high credit ratings find a bank to endorse a bill so the firm can successfully borrow money through the intermediary that is the bank.



-In most cases, the bank that is endorsing the bills charge an acceptance fee that is usually a percentage of the bill, as it is taking on risk of the borrowing firm defaulting.

Pricing coupon paying securities

-commonly used by both governments and corporate entities, where fixed bonds are typically paid every six months with the repayment of face value at maturity. Thus, the interest accounted for are the coupons paid at each interval.

❖ **Example:** Consider a bond which pays an annual coupon rate of 10% with 5 years to maturity and a face value of \$1,000. If the bond has a yield to maturity of 8% what price should it be selling for today? Recompute the price assuming that the coupons are paid semi-annually. What would be the bond's price if it did not pay any coupons?

❖ Price of bond with *annual* coupon payments...

❖ $P_0 = 100[(1 - 1.08^{-5})/0.08] + 1000/1.08^5 = \$1,079.85$

❖ Note that the bond is selling at a **premium** of \$79.85 (= 1079.85 - 1000) to the face value as the **YTM < Coupon rate**

-When you are given the annual coupon rate, the face value and what it is selling today and you want to find out the yield to maturity, the formula can be flipped to discover the yield of the coupon bond.

- ❖ **Example:** Consider a bond which pays an annual coupon rate of 10% with 1 year remaining to maturity and a face value of \$1,000. If it is selling for \$982 today what is its yield to maturity?
- ❖ $P_0 = 982 = (100 + 1000)/(1 + k_d)$
- ❖ $k_d = (100 + 1000)/982 - 1 = 12.0\%$
- ❖ The bond is selling at a **discount** of \$18 to the face value because the **YTM > Coupon rate**

Premium bonds vs discount bonds

- the yield to maturity = coupon rate, then price = face value thus the bond is selling at par
- When the yield to maturity is lower than the coupon rate, thus the price is greater than the face value, the bond is selling at a premium.
- When the yield to maturity is greater than the coupon rate, the price is lower than the face value and thus the bond is selling at a discount. Thus, the coupon rate is too low and the investors are not being compensated for their money.

Price sensitivity to interest rate changes

- Prices of the longer maturity bonds are more sensitive to changes in interest rates as there is a greater chance and thus risk to interest rate changes in the uncertain future.
- E.g two bonds with both annual coupon rate of 10%, Bond A matures in 2 years while Bond B matures in 20 years, what will happen if the market rates change to 6%, 8%, 12%, 14%?

- ❖ Both bonds are currently selling at par
- ❖ Price of bonds A and B
 - ❖ $P_0^A = 100[(1 - (1 + k_d)^{-2})/k_d] + 1000/(1 + k_d)^2$
 - ❖ $P_0^B = 100[(1 - (1 + k_d)^{-20})/k_d] + 1000/(1 + k_d)^{20}$
- ❖ Effect of market interest rate changes on prices

❖ 6%:	$P_0^A = \$1,073.34$	(+7.33%)	$P_0^B = \$1,458.80$	(+45.88%)
❖ 8%:	$P_0^A = \$1,035.67$	(+3.57%)	$P_0^B = \$1,196.36$	(+19.63%)
❖ 12%:	$P_0^A = \$966.20$	(-3.38%)	$P_0^B = \$850.61$	(-14.94%)
❖ 14%:	$P_0^A = \$934.13$	(-6.59%)	$P_0^B = \$735.07$	(-26.49%)

↙ ↘
 Percentage changes in prices compared to the *original* price of \$1,000 for each bond

-When the rates go down, your bond is more valuable as it has a greater yield compared to the market yield, however when the rates go up, your bond yield is lesser to the market yield and thus other investments are more valuable and thus the value of your bond has decreased.

-The longer term bond will also act more extremely to changes in interest rates as it has a longer life period of yield

Lecture 4

Valuation of equity securities

-Shareholders are at the end of the line as the company must fulfil its legal obligations first of paying tax to the government and debt to its creditors, then only will it be able to pay its shareholder's dividends.

Characteristics of ordinary shares

-The present value of all future expected dividends discounted at the appropriate required discount rate, where k_e is the rate of return required by investors for the time value of the money and the risk associated with the company.

$$P_0 = \sum_{t=1}^N \frac{CF_t}{(1+k_e)^t}$$

-Need to consider dividends and not earnings because using earnings will rest double counting of future dividends as the retained earnings of a company can be utilised in the future for the purpose of dividend distribution.

-One period framework, the stock price is equal to the sum of next period's dividend and the expected price discounted at the appropriate discount rate

$$P_0 = \frac{D_1 + P_1}{1+k_e} \quad \text{and} \quad P_1 = \frac{D_2 + P_2}{1+k_e} \quad \text{and} \quad P_t = \frac{D_{t+1} + P_{t+1}}{1+k_e}$$

Over any period, the expected rate of return (k_e) is...

$$k_e = \frac{D_{t+1} + P_{t+1}}{P_t} - 1 = \frac{D_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

k_e = Expected dividend yield + Expected percent price change

-Pricing ordinary shares we need to discount the proceeds in every year within the future.