FNCE30001 INVESTMENTS

Table of Contents

WEEK 1: Risk Aversion and Returns

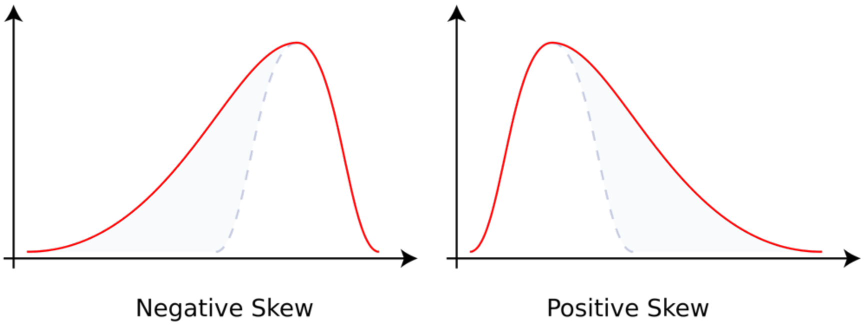
Types of returns:

- Realized
  → after fact, eg at t=1
- Uncertain
  → E(r) at t=0

  $$E(\tilde{r}) = \sum_s p(s)\tilde{r}(s)$$ Where s = given state

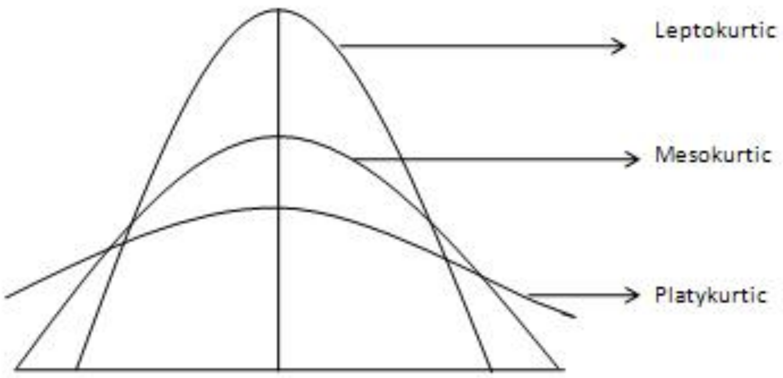
  → Assume r_t be a normal distribution
  - Tractable: symmetric, only need 2 parameters
  - Stable
  - Reasonable: easy to compute probability and confidence intervals

Skewness



- Negative skew (skewed to the left): downside risk is underestimated by standard deviation
- Positive skew (skewed to the right): downside risk is overestimated

Kurtosis



- Leptokurtic distribution: positive kurtosis, distribution is more concentrated around mean, fatter tail
- Platykurtic distribution: negative kurtosis, more dispersed, thin tail

Note: deviation from normality for high frequency returns do NOT come from skewness, but from the more frequent extreme observations.

Portfolio: any collection of investments

- Risk-free asset: returns are certain/can be perfectly estimated
- Risky asset: returns can't be perfectly estimated

Note: for the following formulas, portfolio is represented by 'c', risk-free asset by 'f', and risky asset by 'p'. 'y' represents % of portfolio funds in risky assets.

$$E(\tilde{r}_C) = r_f + y\left[E(\tilde{r}_P) - r_f\right]$$

$$\sigma_C = y\sigma_P$$

2 ways to control a portfolio's risk:

1. Capital allocation → shift funds between risk-free and risky assets
2. Efficient diversification → shift funds across risky assets

In solving problems pertaining to portfolios, we face 2 elements:

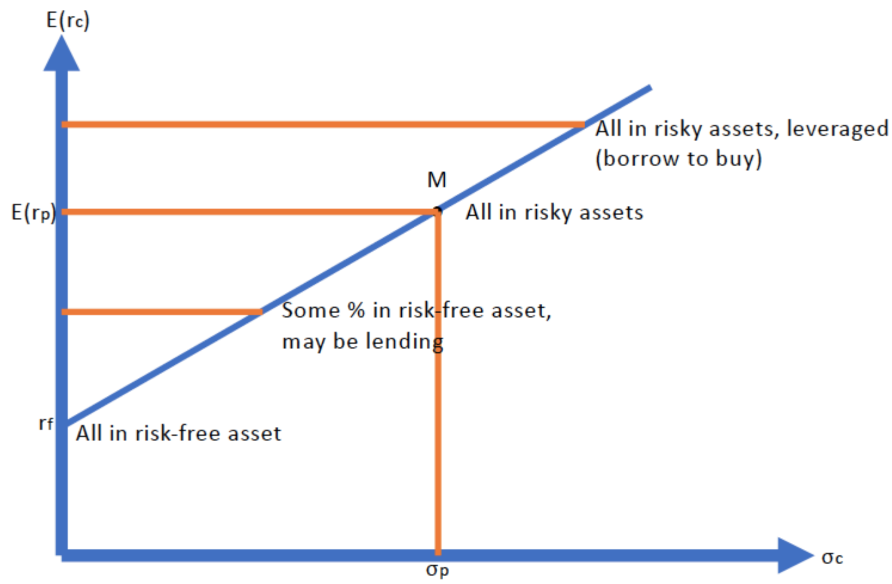
1. Objective → risk-return relationship
2. Subjective → risk-return preference (eg risk aversion/neutrality)

Capital Allocation Line (CAL): line representing feasible investment opportunities

$$E(\tilde{r}_C) = r_f + \frac{E(\tilde{r}_P) - r_f}{\sigma_P}\sigma_C$$

Capital Market Line: line representing feasible investment opportunities of a combination of risk-free assets and the market portfolio

Interpreting CML:



CML is a type of CAL

- If slope changes after point M, it means that the rate of borrowing ≠ rate of lending

Sharpe ratio, represented by S_P, can be interpreted as extra return per unit of risk.

- Slope of CAL
- Reward-to-volatility

- Higher, better!

$$S_P = \frac{E(\tilde{r}_P) - r_f}{\sigma_P}$$

Common assumption: risk-averse investors → represented through utility function at each level of wealth

- Preferred: maximum Expected Utility
- Expected Utility: average over all possible states (s)
- $$U_i = E(\tilde{r}_i) - \frac{1}{2} A\sigma_i^2$$
  Where A represents coefficient of risk aversion
  A = 0 → risk-neutral investor
  A < 0 → risk-loving
  A > 0 → risk-averse investor (MOST LIKELY ENCOUNTERED)

Finding maximum Expected Utility: differentiate in respect to 'y'

Thus,

$$y^* = \frac{E(\tilde{r}_P) - r_f}{A\sigma_P^2}$$

$$y^* = \frac{S_P}{A\sigma_P}$$

where y* represents optimal percentage of risky assets in a portfolio

WEEK 2: Optimal Portfolios

Sources of risk:

- Systematic: market-wide
- Non-systematic/idiosyncratic: firm-specific

Mitigating idiosyncratic risk: efficient diversification!

- Minimize portfolio risk at any given level of Expected Return
- Exploit asset correlation structure:
  Portfolio's standard deviation < weighted average of standard deviation of individual assets
- We want <u>low</u> correlation

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$$

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A\sigma_B}$$