Engineering Economics

Time value of money

The best economic choice will be made through the proper use of the time value of money

What is the difference between cost and price?

The unit of measurement is both money. The price of an item is the exchange ratio of the item in the market. The cost refers to the disadvantage of not having the item – what is takes us to be exchanged for it.

What is money?

It is a medium of exchange. It is used to determine whether an action is economically efficient – quantifies analysis.

What is the primary goal of management?

Maximising profit for the company and its stakeholders, or maximising cost effectiveness in public firms.

What are the basics of cash flow diagrams?

Identifying costs and benefits, converting benefits to the same units (usually money), deriving a timeline (money today is not worth the same as money tomorrow). Central to this analysis is compounding, discounting and economic equivalence

What is the concept of the time value of money?

Money has time value. People prefer current goods over future goods, current money is more valuable than future money. – arises from: aging, uncertainty, interest, investments. This concept needs to be incorporated into our analysis. Therefore we discount future money and consider interest rate for current investments.

What are the two sources of capital needed to make an investment?

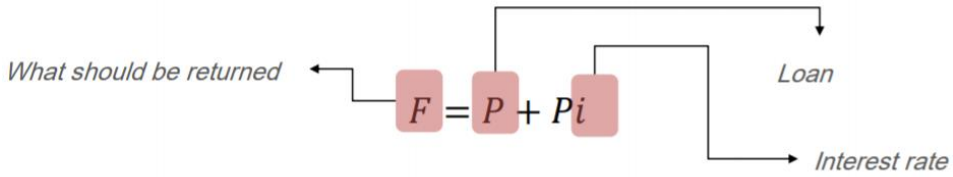
Borrowing from a lender, investor's own fund – in both cases there is a cost associated with the capital. If a capital is obtained from a lender, the cost of capital is the interest rate at which the funds are loaned. If a capital is obtained from the investor's fund, then the cost is called the opportunity cost of capital.

What is the difference between interest rate and inflation?

Inflation is an indication of purchasing power of money, affordability of money you have. E.g. if inflation rate is 3%, what we can afford with $1000 now, we can afford with $1030 a year from now. Even in an economy with zero inflation rate, money has time value. Interest rate depends on: the economic condition (state of the economy), administrative costs, risks associated with a particular investment. Interest rate is defined as the proportion of a loan that is charged as interest to the borrow. It is typically expressed as an annual percentage of the loan outstanding.

P: Present value/money

F: future value/money



Cashflow diagram

What is a cashflow diagram?

It is a visual tool to keep track of all cash incomes and outflows at their appropriate position in time. It helps with the analysis of investment opportunities. The cashflow for any particular time period is the income received during that period minus the expenses incurred during that time period.

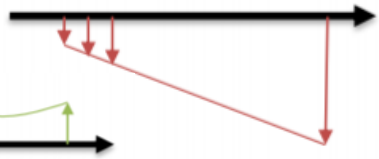
How do you construct a cash flow diagram?

1. **A horizontal line is drawn which represents the length of time (life) of the investment opportunity (project)**
2. **The interest periods are then marked off and labelled below/above the line**
3. **At the extreme left of the time line is time zero – representing the time when the first cash flow is made for this project**
4. **All cash flows are then represented by an arrow corresponding to the position in time which they occurred:**
    a. **Downwards arrows: negative cash flows (expenses exceeding revenues)**
    b. **Upward arrows: positive cash flows**

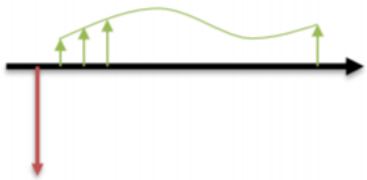
The normal approach is to assume that all investments for a particular year are made at the beginning of the year, while all revenues occur at the end of the year

What are some typical cashflows?

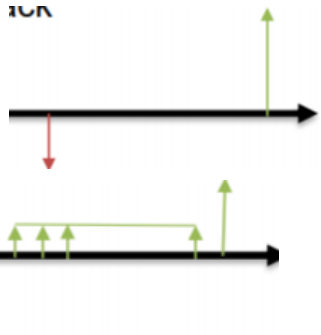
- **Annuity**: uniform series payment e.g. rent, payback of a loan, pension



- **Gradient**: uniform increase of decrease e.g. maintenance cost – low at the beginning, gets higher as the item gets older



- **Investment:** initially a large value of investment, receive it back as income in the future

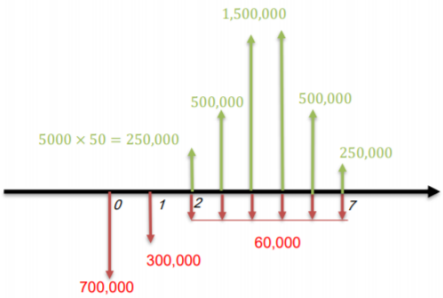


- **Lump-sum pay-back:** putting money in an investment project, do not receive anything until end of project life + interest received
- **Periodic interest payment:** put money into an investment project, receive interest on monthly, yearly basis – at end of project you receive what you invested in the first place

A company has decided to invest in a project to make a product. The initial investment cost will be $1,000,000 to be spread over the first two years with $700,000 in the first year and $300,000 in the second. The plan calls for producing products at the following rates: 5,000 units in year 2; 10,000 in year 3; 30,000 in year 4; 30,000 in year 5; $10,000 in year 6; and $5,000 in year 7.

Products will be sold for $50 each throughout the life of the project and cash operating expenses will be $60,000 per year for years 2 through 7.

Construct a cash flow diagram for the project.



income

t = 2 : 5000 × 50 = 250,000
3 : 10000 × 50 = 500,000
4 : 30000 × 50 = 1,500,000
5 : 30000 × 50 = 1,500,000
6 : 10000 × 50 = 500,000
7 : 5000 × 50 = 250,000

end of time periods

beginning of time periods

Interest formula

Compounding: how an initial investment (P) grows over time

$$F = P(F/P)_{i,n}$$

F given P

depends on interest rate (i) and duration

$$(F/P)_{i,n} = (1+i)^n$$ – duration

interest rate per period

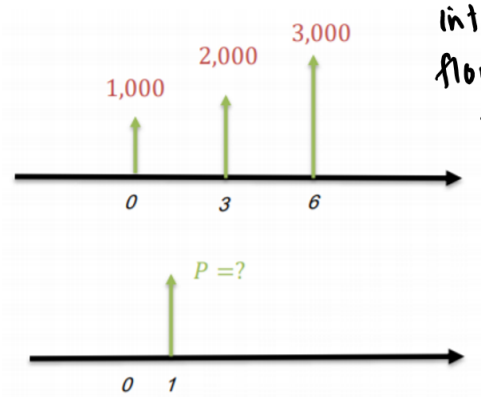
a.y.

Compounding is similar to the quoted interest rate period when interest is charged once in every quoted interest rate period – effective interest rate

Compounding is smaller than the quoted interest rate period when interest is charged more than once in every quoted interest rate period – nominal interest rate

Nominal interest rate – announce interest rate yearly but compounding interest in shorter intervals (e.g. monthly) – the effective yearly interest rate will be higher than the announced nominal interest rate

Calculate the single payment at year 1 of the following 6-year cash diagram if the interest rate is 10% compounded annually.



* Need to convert everything to the same time interval before summing numbers → convert each cash flow back to year 1

i = 10% compounded annually → effective interest rate

$F = P(1+i)^n \quad P = \frac{F}{(1+i)^n}$ at year 1

For \$3000 $\quad P_1 = \frac{3000}{(1.1)^{6-1}} = \frac{3000}{1.1^5} = \$1863$

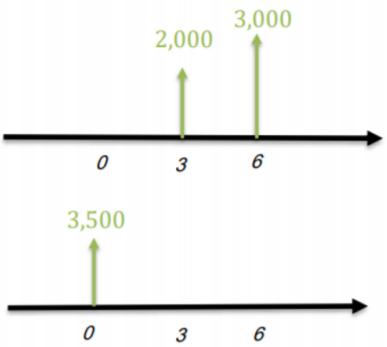
For \$2000 $\quad P_2 = \frac{2000}{1.1^2} = \$1653$

For \$1000 – Future value

$F_3 = 1000(1.1)^1 = \$1100$

$P_{TOT} = P_1 + P_2 + F_3 = \$1863 + \$1653 + \$1100 = \$4616$

Calculate the annual interest rate, for which the following cash flow diagrams are equivalent



→ need common time interval : convert everything to outflow at 0

$F = P(1+i)^n \quad P = \frac{F}{(1+i)^n} \quad P_1 = \frac{3000}{(1+i)^6} + \frac{2000}{(1+i)^3}$ equivalent present value for 2nd cash flow diagram

$P_2 = 3500$ – no interest

want $P_1 = P_2 \rightarrow P_1 - P_2 = 0$

$\frac{3000}{(1+i)^6} + \frac{2000}{(1+i)^3} - 3500 = 0$ can solve quadratically: let $x = (1+i)^3$

or

Guess and check – need an i that gives a value below and above 0

i = 10% $\quad P_1 - P_2 = -303.9$

i = 5% $\quad P_1 - P_2 = 466.3$

linear relationship

$i = 5\% + \frac{466.3}{466.3 + 303.9}(10\% - 5\%) \approx 8\%$

These formulas have all been for single payments

It is often necessary to know the amount of a uniform series payment, A, which would be equivalent to a present sum P

$(P/A)_{i,n}$ -> P given A

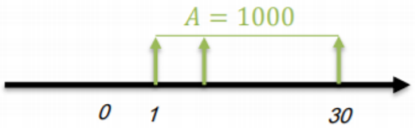
$(A/P)_{i,n}$ -> A given P

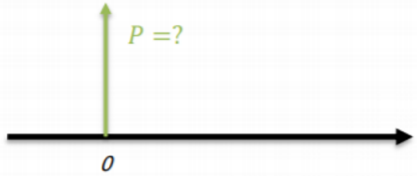
$$P = \frac{[(1+i)^n - 1]}{i(1+i)^n} A \qquad A = \frac{[i(1+i)^n]}{(1+i)^n - 1} P$$

present sum

uniform series payment

An individual wishes to place an amount of money in a savings account and, at the end of one month and for every month thereafter for 30 months, draw out $1,000. What amount must be placed in the account if the interest rate is 12% (nominal rate) compounded monthly?





Nominal interest rate = 12% yearly

∴ effective interest rate compounded monthly = 1%

$P = A(P/A)_{i,n}$

$$(P/A)_{i,n} = \frac{(1+i)^n - 1}{i(1+i)^n} = \frac{(1.01)^{30} - 1}{0.01(1.01)^{30}} = 25.8077$$

∴ $P = 1000 \times 25.808$

$\boxed{P = \$25808}$

*An initial payment of $25808 is equivalent to a uniform payment over 30 months of $1000 -> These two cash flows are equivalent

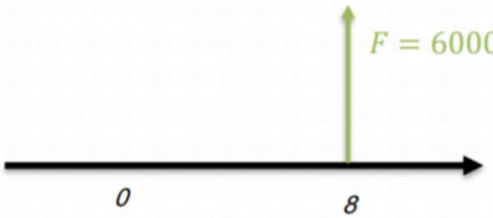
Future value

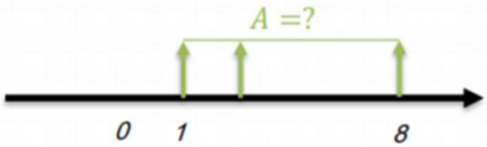
uniform payments

$$(A/F)_{i,n} = \frac{i}{(1+i)^n - 1} \qquad (F/A)_{i,n} = \frac{(1+i)^n - 1}{i} \qquad A = F(A/F)_{i,n}$$

An individual wishes to have $6,000 available after 8 years. If the interest rate is 7% compounded annually, what uniform amount must be deposited at the end of each year?

Future value = F, n = 8, i





$$A = F(A/F)_{i,n} = F \times \frac{i}{(1+i)^n - 1} = \$6000 \times \frac{0.07}{1.07^8 - 1} = \$585$$