Lecture 1

Return

r_1 is the holding period return from time $t = 0$ to $t = 1$

$$r_1 = \frac{P_1 + D_1 - P_0}{P_0}$$

The return is the ex-dividend price P_1, plus any dividend received at time $t = 1$, D_1, minus P_0, the price paid at time $t = 0$, as a fraction of P_0.

The return is either:

- realised (known) at $t = 1$, denoted r, or
- unknown at time $t = 0$, denoted $\tilde{r}$

The expected return is denoted $E(\tilde{r})$.
The $\sim$ indicates that returns are uncertain and has a probability distribution.

For example, let's suppose we know that Qantas is about to announce that it has found a new way to cut fuel costs. Is the price of Qantas shares sure to rise in the future? We don't know because fuel costs aren't the only thing affecting the value of Qantas.

It depends on the economy. We call each future scenario/outcome a "state" s, where states must be mutually exclusive and exhaustive. The return Qantas receives in each state is $\tilde{r}(s)$ and the likelihood of each state depends on the probability distribution $p(s)$, where probabilities must sum up to 1.

The expected return is:

$$E(\tilde{r}) = \sum_s p(s)\tilde{r}(s)$$

Let's say the economy will either be good or bad next year. We make the following forecasts about Qantas stock prices.

Economy	Probability	Qantas Price
Good	0.3	\$1.20
Bad	0.7	\$0.8

If the current price of Qantas is \$1.17

- The **uncertain** future returns, at time $t = 0$ are:

$$\tilde{r}(G) = \frac{(1.20 - 1.17)}{1.17} = 2.56\%$$

$$\tilde{r}(B) = \frac{(0.80 - 1.17)}{1.17} = -31.62\%$$

- The **actual** returns will be either $r = 2.56\%$ or $r = -31.62\%$ at time $t = 1$
- The **expected** return is: $E(\tilde{r}) = 0.3 \times 2.56\% + 0.7 \times -31.62\% = -21.37\%$

Standard deviation

The standard deviation of a return measures the uncertainty of its returns.
This is also referred to as the 'volatility' of returns.

The variance of returns is: $\sigma^2 = \sum_s p(s)[\tilde{r}(s) - E(\tilde{r})]^2$, which is just $E[(\tilde{r} - E(\tilde{r}))^2]$

The standard deviation is:

$$\sigma = \sqrt{\sum_s p(s)[\tilde{r}(s) - E(\tilde{r})]^2} = \sqrt{E[(\tilde{r} - E(\tilde{r}))^2]}$$

In our example,

$$\sigma^2 = 0.3 \times (0.0256 - (-0.2137))^2 + 0.7 \times (-0.3162 - (-0.2137))^2 = 0.0245$$
$$\sigma = 0.1567$$

The Normal distribution with mean μ and volatility σ has these benefits:

- It is tractable — symmetric with only two parameters needed
  $\Rightarrow \sigma$ is the appropriate measure of risk
- Stable: the return on portfolios of normally distributed assets is normally distributed
- Empirically, it is a reasonable first approximation to many assets' returns

It is often useful to scale the original random variable to transform it into a standard normal variable. i.e., a normal variable with mean 0 and standard deviation, $z \sim N(0,1)$
Consider an asset return $\tilde{r}_t \sim N(\mu, \sigma)$. We can transform $\tilde{r}_t$ into a standard normal random variable $\tilde{z} \sim N(0,1)$ by:

$$\tilde{z} = \frac{\tilde{r}_t - \mu}{\sigma}$$

Example: The Australian stock market risk premium from 1925 to 2010 had an annual average of 7.41% and a standard deviation of 18.90%. Assuming that the stock market risk premium is normally distributed, (1)what is the probability that the stock market risk premium will be positive over the next year and (2) what is the 95% confidence interval for the stock market risk premium over the next year?

1. Probability that the stock market risk premium will be positive

$$z' = \frac{0 - 7.41}{18.90} = \frac{-7.41}{18.90} = -0.3921$$
$$P(\tilde{z} > z') = P(\tilde{z} < -z')$$

Excel: $NORMDIST(0.3921, 0, 1, 1) = 65.25\%$