Markowitz model focuses on the variance-covariance analysis to find the optimal weights in the portfolio for the maximum return, but for a large portfolio, it can cause severe estimation error as there are too many inputs and estimations.

CAPM is a single factor model which assumes the market risk is the only systematic risk that explains the asset returns. Many empirical research found CAPM by itself is not a perfect model to work with in reality. It also relies heavily on the assumptions to hold which is almost impossible to test.

A single-factor model of the economy classifies sources of uncertainty as systematic (macroeconomic) factors or firm-specific (microeconomic) factors.

Index model is not a theoretical model. It demonstrates how to do empirical test on the CAPM. You would have learnt this in QM1, QM2 or other Econometrics subjects.

> Treynor & Black ('73)- Under index model "for optimal risky portfolio", a combination of the active and passive portfolio

APT and multifactor models are CAPM variations. Because of the market anomalies, researchers made those multifactor models to further explore factors that can explain asset returns.

Optimal Portfolio Choice

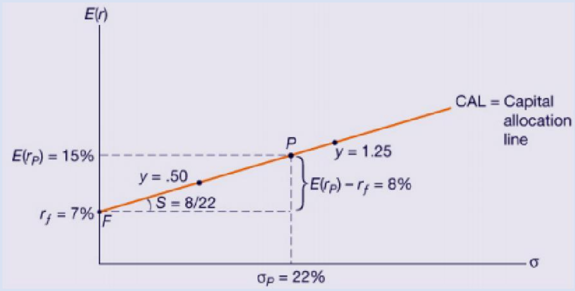
Portfolio	
Optimal portfolio	1. **Capital allocation** between the risky portfolio and risk-free assets: Depends on risk aversion and the risk-return trade-off of the optimal risky portfolio What we did last seminar 2. **Efficient diversification**, or the construction of the optimal risky portfolio ***Which risky assets?*** Domestic shares (BHP), Treasury notes, international shares (Apple, Amazon) But also managed funds, ETFs, real estate (REITs), commodities, etc Also Depends on: ❖ Risk aversion ❖ Investment horizon ❖ Financial goal (growth, income, etc)
Market Risk	**Market risk (systematic risk)**: attributable to economy-wide factors ❖ Domestic business cycle ❖ Inflation ❖ Exchange rate **Unique risk (non-systematic risk)**: firm-specific or idiosyncratic ❖ Fraudulent accounting by the firm ❖ Death of the firm's CEO ❖ ⇒ These risks can be eliminated via diversification
Portfolio's Standard deviation	The portfolio's standard deviation is <u>**smaller**</u> than the weighted average of the individual assets' standard deviations ❖ Even if the covariance between the returns of 2 assets **is positive** ❖ unless the 2 assets are **perfectly positively correlated** (ρ_{ab} = 1)
Capital Allocation Decision – decision how to split funds b/w safe and risky asset	

1. Investment Opp. Set

CAL: is the possible allocation options with rf asset and ONE risky asset (or ONE portfolio of risky asset)

Efficient Frontier: is the possible allocation options with risky assets

Capital Allocation Line (CAL) – the graph of investment opp. set



$$E(\tilde{r}_C) = r_f + \sigma_C \frac{E(\tilde{r}_P) - r_f}{\sigma_P}$$

If P=M (a broad market index)
⇨ Capital Market Line

Sharpe Ratio (reward-to-volatility) – measures the performance

$\dfrac{E(\tilde{r}) - r_f}{\sigma}$; higher ratio indicates better performance

2. Asset Allocation "subjective" element i.e., personal preference

Utility function

Risk Aversion- prefer less risk for the same E(r)

We assume that people maximize Expected Utility E(U)

➔ "Expected": average over all possible future states

Utility Function – ***represents*** *a person's preferences*

$$U_i = E(\tilde{r}_i) - \frac{1}{2}A\sigma_i^2$$

A = "coefficient of risk aversion" (higher, more averse)

A>0: risk-averse; A=0: risk-neutral; A<0: risk-loving

➔ Utility increases with expected returns and (A>0) decrease with risk (more attractive risk-return profiles deliver higher utility)

3. Asset Allocation "Objective" element i.e., the risk-return combination available to the investor

- Which of the many possible portfolios on the CAL to choose?

Utility Maximisation - *that* ***maximizes the investor's utility***

$$\max_y U = E(\tilde{r}_C) - \frac{1}{2}A\sigma_C^2 = r_f + y\left(E(\tilde{r}_P) - r_f\right) - \frac{1}{2}Ay^2\sigma_P^2$$

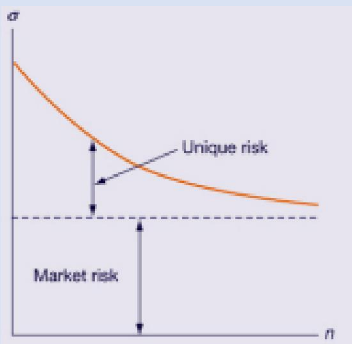
differentiate U w.r.t. y, set the derivative equal to 0, and solve for y to get: y* - optimal proportion of funds in risky portfolio

**AND we used the definition of the Sharpe ratio S_P of portfolio

$$y^* = \frac{E(\tilde{r}_P) - r_f}{A\sigma_P^2} = \frac{S_P}{A\sigma_P}$$

"Markowitz model" aim for Efficient diversification – shift funds across assets only

Portfolio Diversification



B Market and unique risk

To construct an optimal risky portfolio

Efficient diversification: exploit asset correlation structure to minimize portfolio risk for each target expected return. (i.e., "when σ^2_p falls while $E(r_p)$ rises")

Limits of diversification

❖ Average covariance (σ_{ij}) = 0 -> σp^2 (portfolio variance) can be drive to 0

❖ Average covariance (σ_{ij}) > 0 -> σp^2 remains positive

❖ economy-wide risk factors impart positive correlation among stock returns

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<td>Combing Investment</td>
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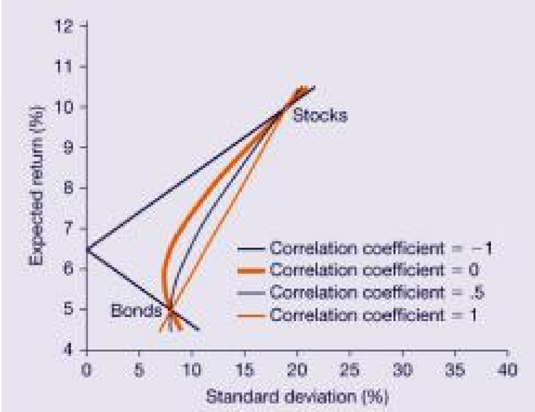

- ρ_{ij} = -1, risk free portfolio
- ρ_{ij} = +1, no benefit
- ρ_{ij} ≠ |1|, diversification benefits improve & the curve "bends backward"

the lower the correlation , the smaller the variance of the 'minimum-variance' portfolio
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<td>
Minimum-Variance Portfolio

CAL: is the possible allocation options with risky free asset and ONE risky asset (or ONE portfolio of risky asset)

Efficient Frontier: is the possible allocation options with risky assets
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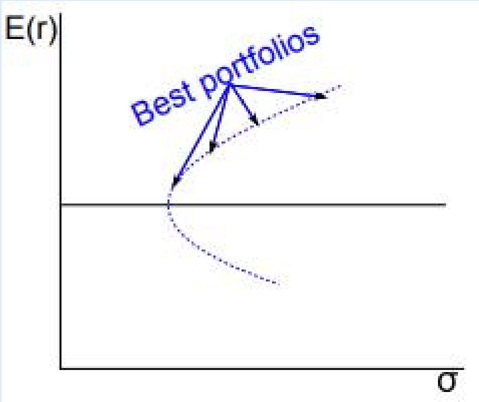
- the lowest variance ("minimum-variance") portfolio for a particular E(r)

Minimum-Variance Frontier (the graph is parabola)
Is the combination b/w two risky asset



$$\min_{w_1,w_2,\ldots,w_N} \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

$$E(r_p) = \sum_{i=1}^{N} w_i E(r_i) = \bar{r}, \quad \sum_{i=1}^{N} w_i = 1.$$

Global min variance portfolio: portfolio with the min possible variance out of all risky portfolio

Efficient frontier:
Best returns for each possible level of risk (upper half)

When N is large

- individual covariance is $\frac{N^2-N}{2}$; if n=100, we need 5050 terms in Excel
- THUS we use Matrix Multiplication (not examinable)

To find M-V frontier, the most important is depends on the different proportion of each risky asset in <u>that</u> portfolio. i.e., that means we are finding the M-V in respect of the possible combination of risky Asset
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<td>Asset Allocation with Constraints</td>
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You may want to constrain the portfolio weights to reflect strategies on:

- No short sales
- Limit exposure to certain asset classes due to transaction costs, liquidity, or uncertainty about estimates:
- No borrowing or lending
_e.g., in optimal risky Portfolio, Wb can't go below 0 or above 1
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