Area of Study #1 - Linear Programming

Lecture 1 - Business Analytics and Linear Programming

Business Analytics

- Use of various data and statistical analysis to help managers gain improved insights about their business operation and make better, fact-based decisions.
  - Eg. using tools such as Microsoft Excel, SPSS

Three Different Elements of Business Analytics

- Descriptive
  - Using summaries of information about the firm to help understand and analyse past and current performance to make informed decisions.
  - Involves categorising, characterising, consolidating, and classifying data
- Predictive
  - Using data about past performance to predict the future based on patterns in relationships in these data, and then extrapolating relationships forward in time.
- Prescriptive
  - Use optimisation to identify the best alternative to minimise or maximise a single or multiple objectives. The mathematical and statistical techniques of predictive analysis can also be combined with optimisation to make decisions that take into consideration the uncertainty in the data.

Linear Programming

- Using mathematical techniques to solve problems pertaining to allocation of resources - eg. machinery, labour, money, time and materials. These resources may be used to produce products or services.

Elements of Linear Programming

- A set of decision variables (x_1 , x_2 ...)
  - Which are used to stand for the quantity of each product or service used
- An objective function
  - Which measures the extent to which alternative feasible decisions achieve the aim being pursued

- A set of constraints
    - Which define in mathematical terms the values of the decision variable that are feasible.

General Form of Linear Programming Model

$$\text{Max or Min} \quad c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$\begin{aligned}\text{Subject to:} \quad & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \\ & a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ & a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \\ & \text{All } x_1, \ldots, x_n \geq 0.\end{aligned}$$

$c_1, \ldots, c_n$:	Objective function coefficients
$a_{11}, \ldots, a_{mn}$:	Technical coefficients
$b_1, \ldots, b_m$:	Right-hand-sides (RHS)
$x_1, \ldots, x_n$:	Decision variables

Assumptions of the Linear Programming Model

- Deterministic model
    - All ci, aij, bj (for i =1, …, m; j=1, …, m) are known with certainty (everything is known with certainty - all constraints, technical coefficients (eg. costs))
- Implications of linearity
    - **Divisibility**: All variables are continuous. For example, we could produce whole tons of products or proportions of tons.
    - **Proportionality**: Value of the function is in direct proportion to the values of the decision variables. For example, if we increase the cost per unit shipped by 10%, then we will increase the total costs of shipments by 10%.

LP Application Type #1- Profit Maximisation Problem

The Shader Electronics Company produces two products:
(1) the Shader xpod, a portable music player, and
(2) the Shader BlueBerry, an internet-connected colour telephone.

The production process for each product is similar in that both require a certain number of hours of electronic work and a certain number of labour-hours in the assembly department. Each x-pod takes 4 hours of electronic work and 2 hours in the assembly shop. Each BlueBerry requires 3 hours in electronics and 1 hour in assembly.

During the current production period, 240 hours of electronic time are available, and 100 hours of assembly department time are available. Each x-pod sold yields a profit of $7; each BlueBerry produced can be sold for a $5 profit. Shader's problem is to determine the best possible combination of x-pods and BlueBerrys to manufacture to reach the maximum profit

Step 1: Fully understand the managerial or optimisation problem being faced.

	Hours required to produce one unit		
Department	x-pod	BlueBerrys	Available hours
Electronic	4	3	240
Assembly	2	1	100
Profit per unit	$7	$5	

How many x-pods and BlueBerrys should Shader manufacture to reach the maximum profit, while not exceeding the limited amount of electronic and assembly time?

Step 2: Define the decision variables.

x_1 = number of x-pods to be produced
x_2 = number of BlueBerrys to be produced

Step 3: Identify the objective function
Maximise profit or z = $7x_1 + 5x_2$

Step 4: Identify the constraints

$4x_1 + 3x_2 \leq 240$ (Electronic department resource constraint)

$2x_1 + 1x_2 \leq 100$ (Assembly department resource constraint)
x_1 and $x_2 \geq 0$ (non-negativity)

Problem Formulation

Maximise profit or z= $7x_1 + 5x_2$

Subject to

$4x_1 + 3x_2 \leq 240$
$2x_1 + 1x_2 \leq 100$
x_1 and $x_2 \geq 0$

This is a linear programming model as:

- Objective function is linear
- All functions on left-hand side of the constraints

<u>Two methods of solving the problem:</u>

Method #1 - Graphical Solution Method

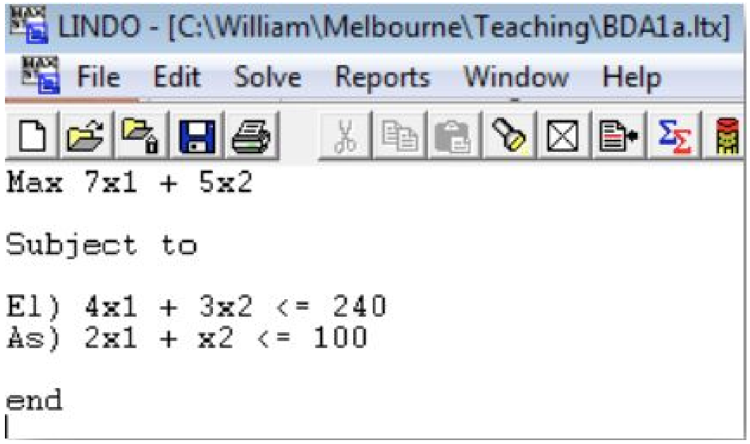
- Plot the solution on a two-dimensional graph

Method #2 - Simplex Method:

- The basis of most LP optimisation software
- Works by finding corner points solutions, evaluating the objective function at each, and stopping when no more improvement to the objective function value (z) is possible.
- Optimisation Software such as Lindo, SAS, etc.
- MS Excel: Solver add-in is needed

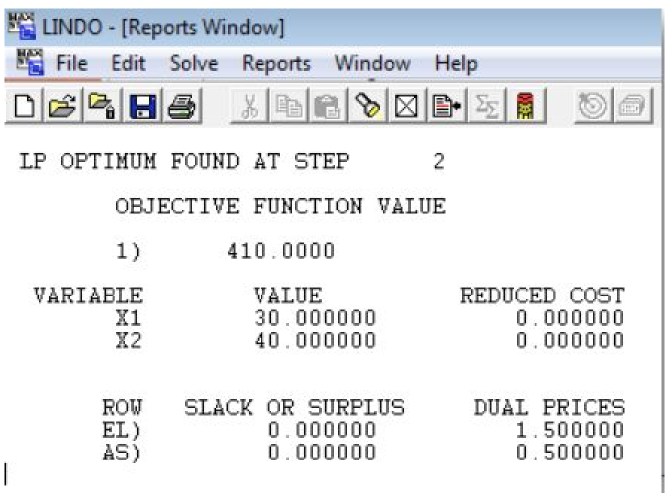
Using Lindo

For the shader problem, the input file is:



When this has been saved, proceed as follows:

- Click “Debug” in “Solve” submenu to debug any errors
- When no more errors, click “Solve” in “Solve” submenu – Answer “No” to the question: “Do Range (Sensitivity) Analysis?”
- To view the solution, Click “Solution” in “Reports” submenu



Linear Programming - Excel Solver

Data Input

- Data Input

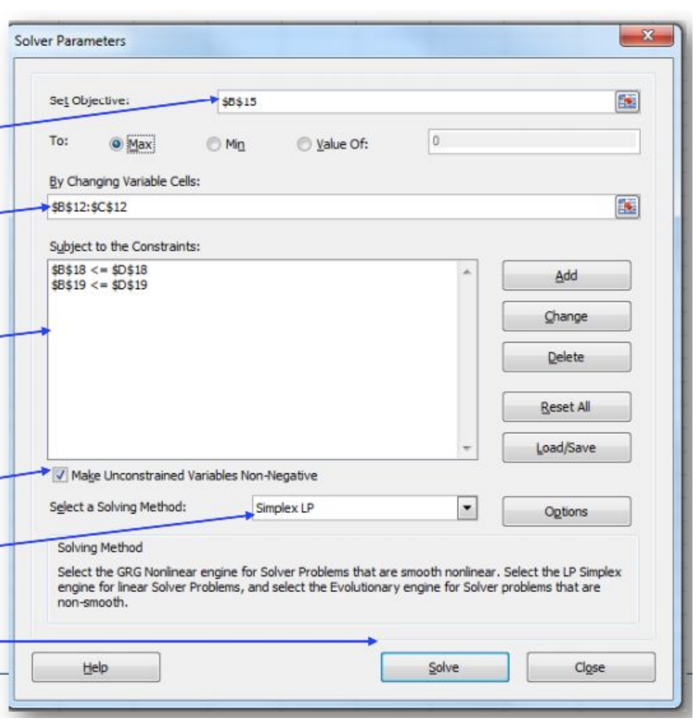
	A	B	C	D
1	The Shader Electronics Company			
2				
3	Hours required per unit	x-pods	BlueBerrys	
4	Electronic	4	3	
5	Assembly	2	1	
6				
7	Profit per unit	7	5	
8				
9	**Optimal Solutions**			
10				
11	Decision variables	x-pods	BlueBerrys	
12	Units produced			
13				
14	Objective function			
15	Profit	0		
16				
17	Constraints	LHS	Inequality	RHS
18	Electronic department resource	0	≤	240
19	Assembly department resource	0	≤	100

=SUMPRODUCT(B7:C7,B12:C12)

=SUMPRODUCT(B4:C4,B12:C12)

=SUMPRODUCT(B5:C5,B12:C12)

- Parameters setting (Data > Solver)



Enter the cell address for the objective function value.

Specify the location of the values for the variables. Solver will put the optimal values here.

Click "Add" to add the constraints to Solver.

Check this box to make the variables non-negative.

Click and select "Simplex LP" from the menu that appears.

Check "Solve" to solve the LP.

- Optimal solution generation

	A	B	C	D
1	The Shader Electronics Company			
2				
3	Hours required per unit	x-pods	BlueBerrys	
4	Electronic	4	3	
5	Assembly	2	1	
6				
7	Profit per unit	7	5	
8				
9	**Optimal Solutions**			
10				
11	Decision variables	x-pods	BlueBerrys	
12	Units produced	30	40	
13				
14	Objective function			
15	Profit	410		
16				
17	Constraints	LHS	Inequality	RHS
18	Electronic department resource	240	≤	240
19	Assembly department resource	100	≤	100

Sensitivity Analysis

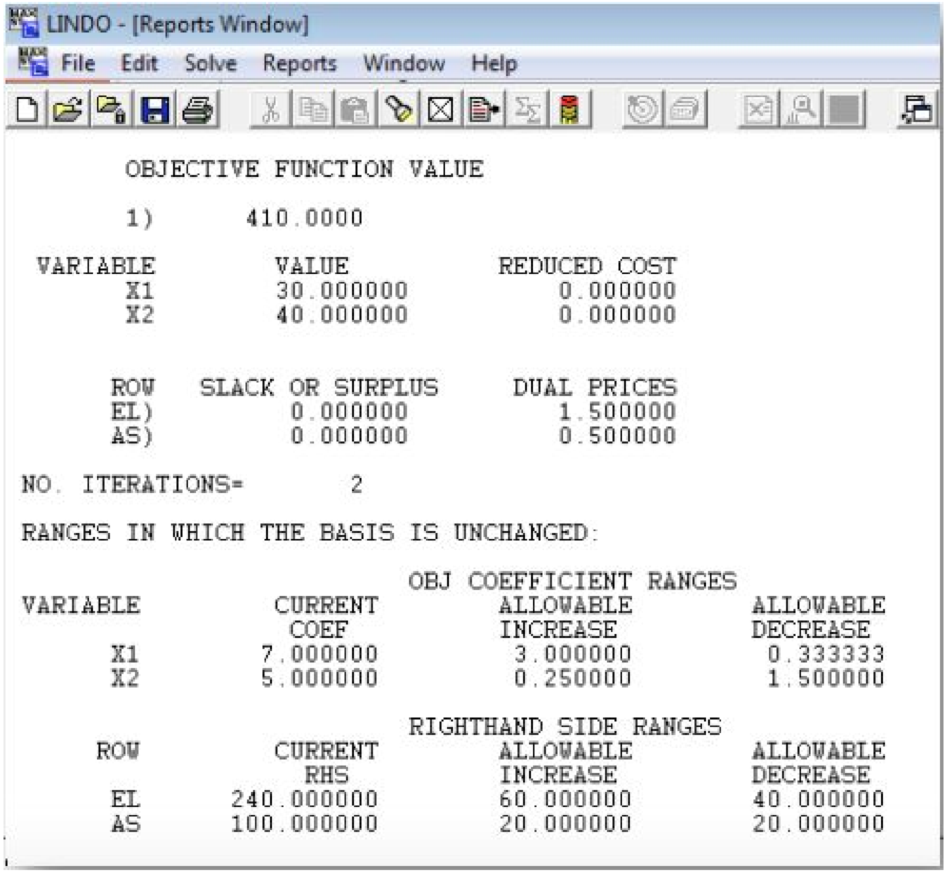
- Reports on the effects of changes in the objective function coefficients, such as
  - Reduced cost: how much the objective function coefficient of each decision variable would have to improve before that variable could assume a positive value in the optimal solution.
  - Allowable increase and decrease: what are the limits that the objective function coefficients can be adjusted without affecting the optimality of the current solution.

Changes in the constraints or right-hand-side (RHS) values

- Shadow price or dual prices (in Lindo): how much objective function value can be improved if the RHS value of a constraint is increased by one unit.
- Allowable increase and decrease: what are the limits that RHS value of a constraint can be adjusted without affecting the validity of the shadow price.

For Lindo

Reports submenu, click Solution, then click Range

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       OBJECTIVE FUNCTION VALUE

       1)      410.0000

 VARIABLE        VALUE          REDUCED COST
       X1        30.000000          0.000000
       X2        40.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
      EL)         0.000000          1.500000
      AS)         0.000000          0.500000

 NO. ITERATIONS=       2

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1        7.000000        3.000000         0.333333
       X2        5.000000        0.250000         1.500000

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
       EL      240.000000       60.000000        40.000000
       AS      100.000000       20.000000        20.000000
```

In the “Solver Report” window, select “Sensitivity” report, and then click “OK”
Select “Sensitivity Report 1” worksheet.

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [BDA1a.xlsx]Sheet1
Report Created: 9/11/2013 1:05:26 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B12	Units produced x-pods	30	0	7	3	0.333333333
C12	Units produced BlueBerrys	40	0	5	0.25	1.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B18	Electronic department resource LHS	240	1.5	240	60	40
B19	Assembly department resource LHS	100	0.5	100	20	20

Interpretation of the Sensitivity Analysis

- You should produce 30 x-pods (D9) and 40 BlueBerrys to maximise profit (D10)
- The final number of hours in the electronic department used is 240 hours (D15), and 100 in the assembly department (D16).
- The constraint of 240 hours in electronic department and 100 in assembly department is shown in F15, F16 respectively.
- Shadow Price (aka. Dual price)
  - Tells you how much your objective value (in this case profit) will change as you obtain/lose additional units of the respective constraint.
  - Ie. Rather than 240 hours in electronic department, if we had 241, we would make extra profit of $1.50.
- Allowable Increase (G15, G16, H15, H16)
  - We can use the shadow price to judge extra profits up to an allowable increase of 60 units, and allowable decrease of 40 units, before we have to rerun a new model.
- Reduced cost
  - Additional profit for every additional unit you produce if there are still resources available.
- Make sure to use the sensitivity report for only objectives within the allowable increase and decrease range: in this case, between 200-300 hours of electronics work and 80-120 hours in assembly work.

LP Application Type #2 - Cost Minimisation Problem

As part of a quality improvement initiative, Consolidated Electronics employees complete a 3-day training program on teaming and a 2-day training program on problem solving. The manager of quality improvement has requested that at least 8 training programs on teaming and at least 10 training programs on problem solving be offered during the next 3 months. In addition, senior-level management has specified that at least 25 training programs must be offered during this period. Consolidated Electronics uses a consultant to teach the training programs. During the next 3 months, the consultant has 84 days of training time available. Each training program on teaming costs $10,000 and each training program on problem solving costs $8,000.

Consolidated Electronics problem is to determine the number of training programs on teaming and the number of training programs on problem solving that should be offered in order to minimise the total cost.

Step 1: Fully understand the managerial or optimisation problem being faced

	Number of training programs being offered		
	Teaming	Problem Solving	
Consultant	3 days	2 days	84 days
Manager 1	1	-	8 program
Manager 2	-	1	10 programs
Senior-Level	1	1	25 programs
Cost per program	$10,000	$8,000	

How many training programs of teaming and problem solving to offer to reach the minimum total cost, while not exceeding the available time of the consultant and meeting the minimum training requirements?

Step 2: Define the decision variables

x_1 = number of training programs on teaming to be offered
x_2 = number of training programs on problem solving to be offered

Step 3: Identify the objective function

Minimise total cost or z = $10000x_1 + 8000x_2$

Step 4: Identify the constraints

$3x_1 + 2x_2 \leq 84$ (Consultant availability constraint)
$x_1 \geq 8$ (Manager requirement 1 constraint)
$x_2 \geq 10$ (Manager requirement 2 constraint)
$x_1 + x_2 \geq 25$ (Senior-level management requirement constraint)
x_1 and $x_2 \geq 0$ (Non-negativity)

Linear Programming - Formulation

Minimise total cost or z = $10000x_1 + 8000x_2$

Subject to
$3x_1 + 2x_2 \leq 84$
$x_1 \geq 8$
$x_2 \geq 10$
$x_1 + x_2 \geq 25$
x_1 and $x_2 \geq 0$