ECOM20001 Econometrics

Lecture Note 2: Probability

One Random Variable

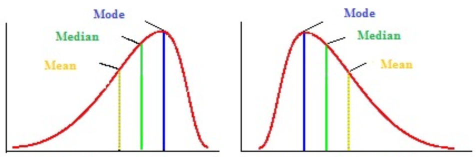
- Random processes and probabilities
- Discrete and continuous random variables
- Probability density functions
- Cumulative density functions
- Describing distributions: mean, variance, skewness, kurtosis

Random processes: flipping a coin, rolling a 6-sided dice

Discrete: discrete set of values (eg flipping a coin)

Continuous: continuum of values (eg household income)

Probability distribution: P(a ≤ X ≤ b)



Left-Skewed (Negative Skewness) Right-Skewed (Positive Skewness)

	Discrete Outcome: Number of Pokies Venues in an LGA									
	1	2	3	4	5	6	7	8	9	10+
Probability Distribution	10.3	7.4	10.3	13.2	4.4	4.4	5.9	7.3	1.5	35.3
Cumulative Probability Distribution	10.3	17.7	28.0	41.2	45.6	50.0	55.9	63.2	64.7	100.0

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Cumulative distribution: P(X ≤ b)

Rules with summation operators:

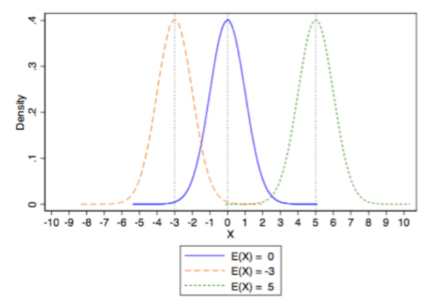
$$\sum_{i=1}^{n} a x_i = a \sum_{i=1}^{n} x_i \qquad \sum_{i=1}^{n} a = n \times a \qquad \sum_{i=1}^{n}(x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$$

E(X) discrete

$$E(X) = \mu_X = p_1 x_1 + p_2 x_2 + \ldots + p_n x_n = \sum_{i=1}^{n} p_i x_i$$

E(X) continuous

$$E(X) = \mu_X = \int_{x_{min}}^{x_{max}} x f(x) dx$$



Rules with E(X)

$$E(aX + b) = aE(X) + b$$

Variance & Standard Deviation:

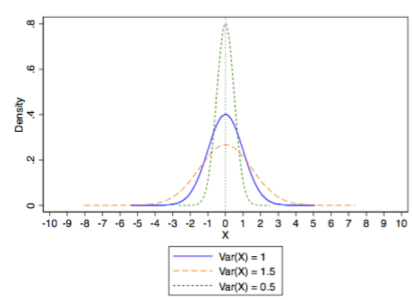
Measures the dispersion or how spread out probability distribution is

Var(X) continuous

$$var(X) = \sigma_X^2 = E[(X - \mu_X)^2] = E(X^2) - \mu_X^2$$

std(X) continuous

$$std(X) = \sigma_X = \sqrt{(\sigma_X^2)}$$



Mean of Linear Function of Random Variable

Y = 6 – 1.5X

E(Y) = E(6-1.5X) = E(6) – 1.5E(X)

$\mu_Y = 6 - 1.5\,\mu_X$

<u>Variance of Linear Function of Random Variable</u>

$$\begin{aligned}\sigma_Y^2 &= E[(Y-\mu_Y)^2)] \\ &= E[(\underbrace{(6-1.5X)}_{Y} - \underbrace{(6-1.5\mu_X)}_{\mu_Y})^2] \\ &= E[(-1.5(X-\mu_X))^2] \\ &= 2.25E[(X-\mu_X)^2] \\ &= 2.25\sigma_X^2\end{aligned}$$

- In general, if random variables X and Y are linearly related according to constants a (intercept) and b (slope):

$$Y = a + bX$$

then the means are related as follows:

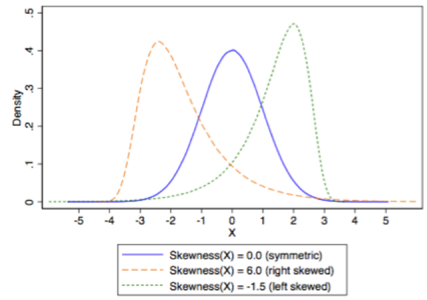
$$\mu_Y = a + b\mu_X$$

and the variances are related as follows:

$$\sigma_Y^2 = b^2\sigma_X^2$$

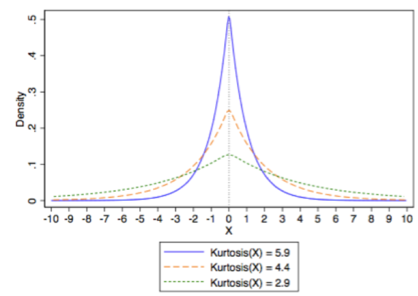
<u>Skewness:</u> symmetry in distribution

$$\text{Skewness} = \frac{E[(Y-\mu_Y)^3]}{\sigma_Y^3}$$



<u>Kurtosis</u>: how much dispersion of distribution is in its tails (tell us something about how much variability of a random variable is driven by more extreme values relative to the mean)

$$\text{Kurtosis} = \frac{E[(Y-\mu_Y)^4]}{\sigma_Y^4}$$



larger value = more infrequent extreme deviations

Two Random Variables

- Joint, marginal, and conditional distributions
- Covariance, correlation, and independence

<u>Joint</u>: P (X = x, Y = y)

	Rain (X=0)	No Rain (X=1)	Total
Long Commute (Y=0)	0.15	0.07	0.22
Short Commute (Y=1)	0.15	0.63	0.78
Total	0.30	0.70	1.00

<u>Marginal</u>:

$$P(Y=y) = \sum_{i=1}^{K} P(X=x_i, Y=y)$$

	Rain (X=0)	No Rain (X=1)	Total
Long Commute (Y=0)	0.15	0.07	0.22
Short Commute (Y=1)	0.15	0.63	0.78
Total	0.30	0.70	1.00

<u>Conditional</u>:

$$P(Y=y|X=x) = \frac{P(X=x, Y=y)}{P(X=x)}$$

	Rain (X=0)	No Rain (X=1)	Total
Long Commute (Y=0)	0.15	0.07	0.22
Short Commute (Y=1)	0.15	0.63	0.78
Total	0.30	0.70	1.00

<u>Covariance</u>: extent to which 2 random variables X and Y move together

$$cov(X,Y) = \sigma_{XY} = E[(X-\mu_x)(Y-\mu_Y)]$$

(+): X and Y move together (-): opposite directions