American call option with dividends can be calculated by finding the payoff a little differently, rather than using the immediate value of c_U, c_N, c_D, c_A and c_B we compare it with the respective value of exercising the option at that point of time. That is, we have a new payoff tree as,

$$
\begin{array}{cccc}
 & & & 35 \\
 & & \max(S_{cum} - K, c_U) & \\
 & \max(S - K, c_A) & & 0 \\
 & & & 0 \\
\max(S - K, c) & & \max(S_{cum} - K, c_N) & \\
 & & & 0 \\
 & \max(S - K, c_B) & & 0 \\
 & & \max(S_{cum} - K, c_D) & \\
 & & & 0
\end{array}
$$

Here we use the S_{cum} because it is a call option, but if it is a put option, we use the S_{ex}, and instead of $\max S_{cum} - K, c$ we use $\max K - S_{ex}, p$.

$$
\begin{array}{cccc}
 & & & 35 \\
 & & \max(20, 19.17) & \\
 & \max(S - K, c_A) & & 0 \\
 & & & 0 \\
\max(S - K, c) & & \max(0, 0) & \\
 & & & 0 \\
 & \max(S - K, c_B) & & 0 \\
 & & \max(0, 0) & \\
 & & & 0
\end{array}
$$

Which means that,

$$
\begin{array}{cccc}
 & & & 35 \\
 & & 20 & \\
 & \max(S - K, c_A) & & 0 \\
 & & & 0 \\
\max(S - K, c) & & 0 & \\
 & & & 0 \\
 & \max(S - K, c_B) & & 0 \\
 & & 0 & \\
 & & & 0
\end{array}
$$

Calculate c_A we use the one-period binomial model again, which we get,

$$c_A = c_U \cdot \theta_U + c_N \cdot \theta_D$$

$$c_A = 10.95$$

Calculate c_B we use the one-period binomial model again, which we get,

$$c_B = c_N \cdot \theta_U + c_D \cdot \theta_D$$

$$c_B = 0$$

			35
		20	
			0
	$\max(5, 10.95)$		
			0
$\max(S - K, c)$		0	
			0
	$\max(0, 0)$		
			0
		0	
			0

Calculate c we use the one-period binomial model again, which we get,

$$c = c_A \cdot \theta_U + c_B \cdot \theta_D$$

$$c = 6$$

$$\max(S - K, c) = \max(0, 6) = 6$$

Conclusion As American call option can be exercised at any time, therefore, we have the maximum payoff when exercising the option at time 2 (before the dividend is paid) and when the stock price goes up twice. This method can be checked by comparing the value of the European (5.75) and American (6), where American call option has a higher price!