SAMPLE MATERIAL

- **Steps for calculating the complete portfolio (Markowitz selection model);**
  1. Specify the return characteristics of all securities (expected returns, variances, covariances)
  2. Establish the risky portfolio (asset allocation);
     a. Calculate the optimal risk portfolio using one of the formulas below depending on if we have total returns or excess returns;
        i. $Optmal\ risky\ portfolio\ weights\ (of\ two\ risky\ assets) = w_A = \frac{[E(r_A) - r_f]\sigma_B^2 - [E(r_B) - r_f]\ Cov(r_A, r_B)}{[E(r_A) - r_f]\sigma_B^2 + [E(r_B) - r_f]\sigma_A^2 - [E(r_A) - r_f + E(r_B) - r_f]\ Cov(R_1, R_2)}\ and\ w_B = 1 - w_A$
     b. Calculate the properties of the portfolio using the weights determined from part A (i) and equations below;
        i. $Portfolio\ expected\ return = E(Rp) = W_1E(R_1) + W_2E(R_2)$
        ii. $Portfolio\ variance = \sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(r_1, r_2)$
  3. Allocate the funds between the risky portfolio and the risk free asset (capital allocation);
     a. Calculate the fraction of the complete portfolio allocated to the risky portfolio and to T-bills (risk free asset) using equation below;
        i. $Determining\ risk\ free\ and\ risky\ weights\ in\ the\ complete\ portfolio = Y^* = \frac{E(r_p) - r_f}{A\sigma_p^2}$
     b. Calculate the share of the complete portfolio invested in each asset and in T-bills.
- **Minimum-variance frontier;** this frontier is a graph of the lowest possible variance that can be attained for a given portfolio expected return.
- **Security Issuance;**
- Primary market –
  - *Public offering;* An IPO which may be underwritten by an investment bank.
  - *Private placement;* done privately with largely institutions and wealthy individuals which reduces disclosure requirements but is subject to size limitations.
- Secondary market –
  - *Direct search markets;* Buyers and sellers have to find each other
  - *Brokered market;* Brokers search out buyers and sellers but don't own the asset
  - *Dealer markets;* Dealers have inventories of assets from which they buy and sell
  - *Auction markets;* Traders converge either physically or electronically to trade in one place.
- **The expected return-beta relationship;**
  - $E(R_i)\ (single\ index\ model) = \alpha_i + \beta_iE(R_M)$
- **Risk and covariance in the single index model;**
  - Total risk = systematic risk + firm specific risk
    - $\sigma_i^2(single\ index\ model) = \beta^2\sigma_M^2 + \sigma^2(e)$
  - Covariance = Product of betas x market index risk
    - $cov\ (r_ir_j) = \beta_i\beta_j\sigma_M^2$
  - Correlation = product of correlations with the market index
    - $Corr(r_ir_j)(single\ index\ model) = \frac{\beta_i\beta_j\sigma_M^2}{\sigma_i\sigma_j} = \frac{\beta_i\sigma_M^2\beta_j\sigma_M^2}{\sigma_i\sigma_M\sigma_j\sigma_M} = Corr(r_ir_M) \times Corr(r_jr_M)$
- **Interest rate sensitivity of bond prices;**
  1. Bond prices and yields are inversely related; as yields increase, bond prices fall; as yields fall, bond prices rise.
  2. Convexity; An increase in a bond's yield to maturity results in a smaller price change than a decrease in yield of equal magnitude.
  3. Prices of long term bonds tend to be more sensitive to interest rate changes than prices of short term bonds
  4. The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increases. In other words, interest rate risk is less than proportional to bond maturity
  5. Interest rate risk is inversely related to the bonds coupon rate. Prices of low coupon bonds are more sensitive to changes in interest rates than prices of high-coupon bonds.
  6. The sensitivity of a bonds price to a change in its yield is inversely related to the yield to maturity at which the bond currently is selling.
- **Summary of optimisation procedure;**
  1. Compute the initial position of each security in the active portfolio as $Initial\ weights\ in\ active\ portfolio = w_i^0 = \frac{\alpha_i}{\sigma^2(e_i)}$
  2. Scale those initial positions to force portfolio weights to sum to 1 by dividing by their sum, that is, $Scaled\ weights\ in\ active\ portfolio\ w_i = \frac{w_i^0}{\sum_{i=1}^n w_i^0}$
  3. Compute the alpha of the active portfolio: $Alpha\ of\ active\ portfolio = \alpha_A = \sum_{i=1}^n w_i\alpha_i$
  4. Compute the residual variance of the active portfolio: $Varianceof\ active\ portfolio = \sigma^2(e_A) = \sum_{i=1}^n w_i^2\ \sigma^2(e_i)$

5. Compute the initial position in the active portfolio: $Initial\ position\ in\ active\ portfolio = w_A^0 = \frac{\alpha_A / \sigma^2(e_A)}{E(R_M) / \sigma_M^2}$
6. Compute the beta of the active portfolio: $Active\ portfolio\ beta = \beta_A = \sum_{i=1}^{n} w_i\beta_i$
7. Adjust the initial position in the active portfolio (because we assumed beta was 1): $Adjusted\ initial\ position\ in\ active\ portfolio = w_A^* = \frac{w_A^0}{1+(1-\beta_A)w_A^0}$
8. Note: the optimal risky portfolio now has weights: $w_M^* = 1 - w_A^*;\ w_i^* = w_A^* w_i$
9. Calculate the risk premium of the optimal risky portfolio from the risk premium of the index portfolio and the alpha of the active portfolio: $Risk\ premium\ of\ optimal\ risky\ portfolio = E(R_P) = (w_M^* + w_A^*\beta_A)E(R_M) + w_A^*\alpha_A$ Notice that the beta of the risky portfolio is $w_M^* + w_A^*\beta_A$ because the beta of the index portfolio is 1.
10. Compute the variance of the optimal risky portfolio from the variance of the index portfolio and the residual variance of the active portfolio: $Variance\ of\ optimal\ risky\ portfolio = \sigma_P^2 = (w_M^* + w_A^*\beta_A)^2\sigma_M^2 + [w_A^*\sigma(e_A)]^2$

- **Behavioural finance;** the premise of behavioural finance is that conventional financial theory ignores how real people make decisions. The irrationalities of people fall into two broad categories;
  1. Information processing - Investors do not always process information correctly and therefore infer incorrect probability distributions about future rates of return. Four important instances of information processing error include;
     - *Forecasting error;* people give too much weight to recent experience compared to prior beliefs when making forecasts (sometimes dubbed memory bias) and tend to make forecasts that are too extreme given the uncertainty inherent in the information.
     - *Overconfidence;* people tend to overestimate the precision of their beliefs or forecasts and the tend to overestimate their abilities. This relates to the persistence of active management (overconfident) despite its lack of ability to beat the market.
     - *Conservatism;* investors are too slow (too conservative) in updating their beliefs in response to new evidence.
     - *Sample size neglect and representatives;* the notion of representativeness bias holds that people commonly do not take into account the size of a sample, acting as if a small sample is just as representative of a population as a large one. This leads to people inferring patterns too quickly and extrapolating trends too far into the future.
  2. Behavioural biases - Even when investors are given a probability distribution of returns, they often make inconsistent or systematically suboptimal decisions. Instances of behavioural biases include;
     - *Framing;* decisions seem to be affected by how choices are framed as the wording can trigger different responses despite the underlying probabilities being the same (eg 20% of you will fail vs 80% of you will pass)
     - *Mental accounting;* mental accounting is specific form of framing in which people segregate certain decisions. An example of this is the *house money effect*, in which gamblers are more willing to accept new bets if they are currently ahead because they have segregated this new bet to use their "winnings" rather than the money they entered with.
     - *Regret avoidance;* psychologists have found that individuals who make decisions that turn out badly have more regret (blame themselves more) when the decision was more unconventional. In a investment sense buying a blue chip portfolio that goes down will inflict less regret then a investment in a startup that goes down an equal amount.
     - *Disposition effect;* refers to the tendency for investors to hold onto losing investments as they are unwilling to realise their losses.