Topic 1: Introduction/ Terminology

Terminology	
Derivative Security	A derivative's value derives from the underlying security A derivative is a financial contract with value derived from another asset Derivative Trade in zero net supply market and Is a zero-sum game **Futures / Forwards** - Contracts for future delivery with future payment - Obligations for both parties at future time - Same value if interest rates are deterministic - Both fix a transaction price at some point in the future - Both parties are obliged to transact in the future **Options** - Contracts for future delivery with immediate payment - Obligation only for one party at future time - Right for other party at future time - Time value and intrinsic value
Risk	Derivatives themselves are risky When matched with other risky investments, resulting portfolios can be less risky ¬ Exchange rate risk ¬ Interest rate risk ¬ Commodity price risk Credit risk (including settlement risk) / Market risk (or price risk) / Operational risk (also known as operations risk) / Legal risk / Liquidity risk (1. Related to specific products; 2. Related to the general funding of the institution's operations risk "**Speculators**": want more risk - Not actually want the risk itself; they want exposure to the underlying risk - E.g., if I think corn price is too high, I probably short sell corn BUT, I can sell a futures contract to expose me to corn price changes "**Hedging**": want to reduce risk - 'Hedge your bets' to protect loss by supporting more than one possible result or both sides in a competition - 'hedge funds' try to take two sides of the same gamble i.e.., "market-neutral" exposure

Topic 2: Arbitrage and Trade

Practicing Arbitrage Opportunity	
Finding an Arbitrage	We need one asset to remove one cash flow - More generally, we need one asset to remove one source of uncertainty - Will be crucial in option pricing as well Getting all of our cash flows to one point in time is the goal
No Arbitrage Pricing	*the 'correct' price for Bonds A and B are the prices that remove any arbitrage opp. **The Payoff Table Approach** 1. Understand the problem, using numbers if necessary. [redacted] 2. Interpret cash flows. [redacted] 3. Gather variables to one side of the equality and set the net cash flows at one of the dates to zero. [redacted]
Q4.1	- Bond 1: Two year, 5% (annual) coupon, $1000 par, price = $1000 - Borrow from ANZ at 9% or save at 5%

Bond	0	1	2
1 SELL	+1000	-50	-1050
ANZ save period 1	-1000	+1050	
	0	+1000	-1050
ANZ save period 2		-1000	+1050
Payoff	0	0	0

➔ NO arbitrage opp, as we sell bond 1 at its par value, therefore the yield return is equal to its coupon rate

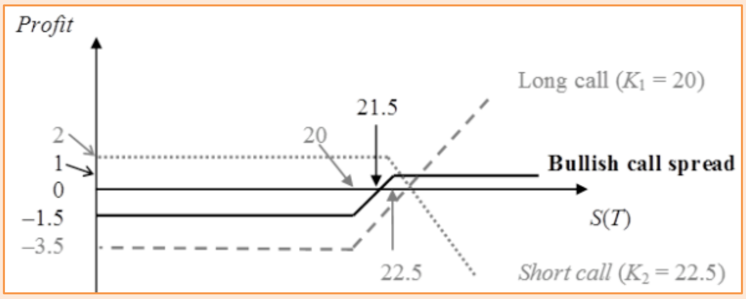
Topic 3: Option Trading and Arbitrage Relations (Strategies and Put-Call Parity)

Bullish Strategies	**Bearish Strategies**
Bullish pay off when the underlying increase in value (all expected price to go up) - Buying stock - Buying a call - Selling a put_ as when price keep down, sold the put would losing more values (as starts at -X and end up to zero)	**Bearish** pay off when the underlying decreases in value (expected price to go down) - Shorting stock - Selling a call - Buying a put ** spread: two exercise prices involved"

"Strategy" ***Not a hedge, just to make some money without causing too much**

Vertical Bullish Spread

Vertical Bullish Spread: payoff more when price are higher



1. Buy call at relatively low strike price Sp1 ($20)
2. Sell call at relatively high Sp2 ($22.5)
Breakeven point $21.5 comes from:

The total premium is 1.5, the payoff of the portfolio for the stock price range of 20 and 22.5 is S-20. So the corresponding profit and loss is S-20-1.5, the break-even point is when the profit is zero, which is when S=21.5.

∴ payoff= whenever price is b/w Sp1 and Sp2

->relatively inexpensive, as cash out and cash in limit your loss (can only loss up to $1.5_difference of call price)

Payoff	**t=0**	**t=T** **$S_t<X_1$**	**t=T** **$X_1<S_T<X_2$**	**t=T** **$S_T>X_2$**
Buy call 1	-C1	0	S_T-X_1	S_T-X_1
Sell call 2	+C2	0	0	(S_T-X_2)*-1 As we sell 1
	+C2-C1	0	S_T-X_1	X_2-X_1

Topic 4&5: Binomial Option Pricing

1. **Hedging Approach**

	Now	**1 Year**	**1 Year**
		S = $40	**S = $65**
Long call	$-X_0$	$+X_D$	$+X_U$
Buy Shares	$-m * S_0$	$+m * S_D$	$+m * S_U$
Borrow	+b	-b*(1+R)	-b*(1+R)
Total	$-X_0 - m*S_0+b$	$+X_D + m*S_D - b*(1+R)$	$+X_U + m*S_U - b*(1+R)$

- Choose m and b so both Totals = 0 in year 1; i.e., set two equation equal

OR//

	[redacted] $m = -\frac{X_U - X_D}{S_U - S_D}; \quad b = \frac{mS_D + X_D}{1+r} = \frac{mS_U + X_U}{1+r}$ [redacted] Buying a bond [redacted] lending money [redacted] lower the better Selling a bond [redacted] borrowing money [redacted] higher the better [redacted] • **Then apply "law of one price"** **Long call = Long Stock + Borrow** [redacted] Borrow [redacted] buy the bond [redacted] P < 83.33, arbitrage Lend [redacted] sell the bond [redacted] P > $100, arbitrage [redacted] • **Then apply "no arbitrage" pricing** **Long 1 call + (Short) m Stock+ (Lend) $b**
2. State prices approach	[redacted] $\theta_U = \frac{S_0 - \frac{S_D}{1+r}}{S_U - S_D}$ $\theta_D = \frac{1}{1+r} - \theta_U$ [redacted]

Topic 6: Monte Carlo

Basics	❖ An "Expectation" is like an average ➢ And can do on functions too; e.g., E[f(X)] is expectation of f(X) ❖ Think Risk-neutral prob as a normal distribution (mean return is rf rate) ➢ Calc X(S), i.e., derivative payoff for every possible outcome S ➢ AND use, normal distribution to get E[X(S)] However, there's infinite numbers of X(S), and maths is undoable ➔ Generate a bunch of random variables and calc the average payoff to the derivative AND discount at risk-free rate
Monte Carlo Valuation - **Good at keeping track of where things have been on the way to**	[redacted] risk-neutral [redacted] future values [redacted] payoff [redacted]

ending stock price	[redacted] discounted [redacted] averaged [redacted] Pros: [redacted] Cons: [redacted]

Lookback Option – an exotic option that's neither call or put

Brief: ➢ *Address the bad things of American Options. Where once you cashed out, you gave up all the future possibilities you could have made*	❖ If the stock price ended up low and came back, you are allowed to buy that relatively low price ❖ If the stock price falls, and keeps falling, you are allowed to sell at the relatively high price ➔ Allows you to transact @ MIN and MAX of stock price over a period of time A (floating) Lookback put pays off: ➢ max = S_{max}- S_T [payoff: diff in max P and ending P] • *If the ending P= max P, you earn nothing, payoff=0* • S_{MAX}= max stock price over the next period ➢ LC payoff max = S_T- S_{min}
Price of Lookback Option	❖ Assume we know how to simulate a lognormal distribution to generate stock price paths, then can use Monte Carlo to compute the value $$\overline{P_0} = \frac{1}{N}\sum_{i=1}^{n}\frac{S_{i;max} - S_{i;T}}{e^{rt}}$$ ➢ However, this is NOT true price, only an approximation

Monte Carlo Properties - How far is the Monte Carlo estimate from the 'true' price?

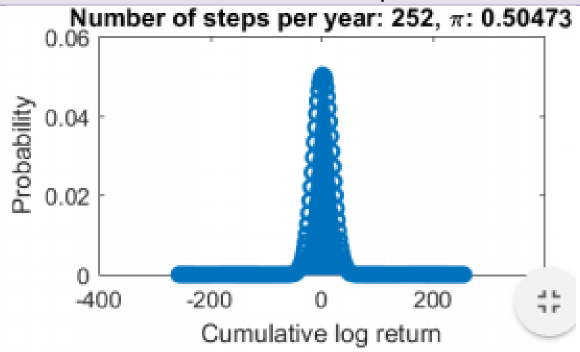
1. Law of Large Numbers (LLN)	[redacted] **infinite number of draws** [redacted]

Topic 8&9: Black-Scholes-Merton

Assumption: **The price path is continuous** ****if believe all, then BSM gives TRUE answer!** **Return are normally distributed**	A1. No market frictions. A2. No credit risk. A3. Competitive and well-functioning markets. A4. No intermediate cash flows. A5. No arbitrage opportunities. A6. No interest rate uncertainty. A7. Trading takes place continuously in time. A8. The stock price follows a lognormal probability distribution. ❖ The price path of the underlying asset is continuous; i.e., no jumps. ❖ The return on the underlying asset is independently distributed through time and the variance is nonstochastic. (We saw last week that the binomial model could handle some situations of stochastic volatility) ❖ The risk-free rate is non-stochastic (i.e., know how to find it)

- We have a European Option with Fixed maturity T and σ and r are constant over time, while is continuous trading
  ➔ BSM is correct, and binomial is only an approx.
- If assumption violated, binomial is approx. right, but BSM is incorrect
  ➔ For the American option or non-continuous trading

one year in Finance is 252 days



From binomial into normal;
- Know the prob to exercise (When price < k)
- Know the prob to not exercise (When price > k)

❖ Prob of Exercise = prob of stock price above K (strike)

$$P[Exercise] = P[S(T) > K]$$

❖ Expected Cost of Exercise =

$$K * P[Exercise] * e^{-rT} + 0 * \ P[Exercise] * \ e^{-rT}$$
$$= K * P[Exercise] * e^{rT} \ = P[\ln S(T) > \ln K]$$

❖ **ASSUME 1:** $\boldsymbol{lnS(T) = lnS(0) + r - \frac{1}{2}\sigma^2) * T + \sigma\sqrt{T} * Z}$

❖ **ASSUME 2:** $Z \sim N(0,1)$

➔ **Solve 1 for Z, then look up the prob form a normal table!**

Using BSM to Hedge!!!

Delta
Gamma
Delta-Gamma Hedge

Delta: hedge option

❖ Delta gives the number of shares to replicate a long call

$$m = -\Delta = -\frac{Xu - Xd}{Su - Sd} \sim \Delta \text{ in derivative value/ } \Delta \text{ in stock price}$$

$$\frac{dc}{ds} = \Delta = N(d1) = \frac{\ln\frac{S}{K} + (r - \frac{\sigma^2}{2}) * T}{\sigma\sqrt{T}} + \sigma\sqrt{T}\ .$$

➔ How an option change when stock price change (over very small change)

❖ i.e., with a d1=-0.022; N(d1)= 0.4920

➔ when stock increase by $1, the option price would approx. up by 49 cents,

Topic 10: Forwards& future

Forward Pricing

Forward Pricing
- **Cost of carry**

(range of) no-arb. forward price

$$F = Se^{rT}$$

- Does not depend on the future spot price or even its expected value
- Its purely determined by opp. cost/arbitrage

Example:

❖ Spot=$100; borrow @ r=5%; lend @ r=3%; T=6/12; F=$102

❖ Long spot +borrow = short forward; short spot+ lend= long forward

	0	T = 1/2
Long spot	-$100	S(T)
Borrow	$100	-$100*$e^{6/12*0.05}$ = -$102.53
Short Forward	0	$102 – S(T)
Total	0	-$0.53

Borrow/lend @ 5%:

- F=$102.53

Borrow @5%, lend @3%:

- F ≥ $101.51
- F ≤ $102.53

With transaction cost:

- F ≥ 99.24
- F ≤ $104.81

Difference b/w rates and transaction cost widen the range of no-arb. forward prices

Cash-and-carry:

Always losing: yield of loss

Reverse Cash-and-carry

- Reversing the strategy cannot be done, as different r
- ∴ cannot know the price b/w & cannot tell whether it's arb. opp.
- → Cash and carry yields an upper bound on F = $102.53
- → Reverse cash-and-carry yields a lower bound on F=$101.51

If bring in Transaction Cost:

	0	T = 1/2
Short spot	$100	-S(T)
Lend	-$100	$\$100*e^{6/12*0.03} = \$101.51$
Long Forward	0	S(T) - $102
Total	0	-$0.49

- Contract cost $0.25 when initiated
- Stock incrus $1 transaction costs, i.e., buying the spot today costs $101, selling yields $99
  - cash-and-carry: F ≤ $104.81
  - Reverse cash-and-carry: F ≥ $99.24

Future Markets

Margin requirement

- Clearinghouse Monitor Credit Risk by enforcing **Margin Requirement**
- Plus it's not feasible to transfer the 'actual' underlying (goods may be perishable, may not have enough money)

[redacted]