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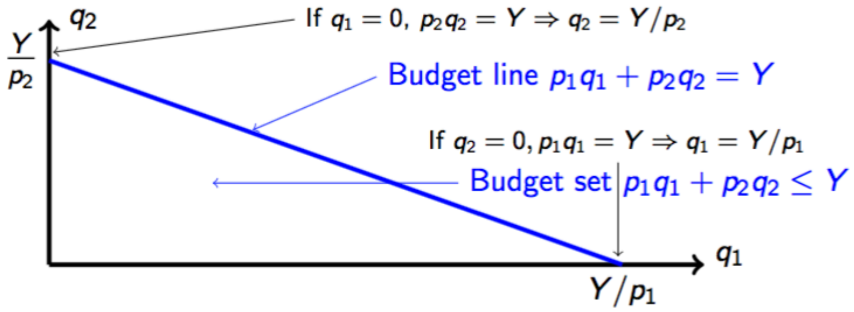
Topic 2: Constrained optimisation

Budget constraint

- Good 1 and good 2 cost money and an agent has a limited amount of it
- A consumer is restricted to choose a consumption bundle $q = (q_1, q_2)$ such that $p_1 q_1 + p_1 q_2 \leq Y$.



Consumer maximisation problem

- Formally, we can write this problem as:

$$\max_{q_1, q_2} u(q_1, q_2)$$
$$\text{subject to } p_1 q_1 + p_2 q_2 \leq Y$$

- Solving it graphically:
  Look for the North-East-most indifference curve, where it touches the budget constraint

What can go wrong?

Satiation

- Consider preferences:

$$u_A(q_1, q_2) = -(q_1 - 1)^2 - (q_2 - 1)^2$$

  Any positive monotonic transformation of a utility function leads to a utility function with the same preferences
  Hence, we can add constant to make the utility positive
- Note that $u_A(q_1, q_2) \leq 0$, and u_A is equal to zero only if $q_1 = q_2 = 1$
- If can afford bundle (1,1), always choose it, because any other budle will give a lower utility

Implication of monotonicity: theorem

- If preferences are monotonic, we can replace inequality $\leq$ with $=$ in the consumer maximisation problem, so $p_1 q_1 + p_1 q_2 = Y$
- **Proof by contradiction**:
  Suppose we cannot replace $\leq$ with =. In other words, $p_1 q_1' + p_1 q_2' = Y' < Y$. Then agent has (Y-Y')>0 income left over, which can be spent on good 1 and good 2 to buy bundle (q_1, q_2).
  - Note that $(q_1, q_2) \gg (q_1', q_2')$. Since we assumed monotonicity of preferences, $(q_1, q_2) \succ (q_1', q_2')$: agent is better off with bundle (q_1, q_2)
  - Note that, by construction, (q_1, q_2) is affordable: $p_1 q_1 + p_1 q_2 = Y$

Topic 7: Political Competition

Modelling decisions

- What is a "party position"?
  - Formal description: party will pick a position as a number in [0,1] interval
- How do voters vote?
  - Each voter knows parties' positions, has one's own positions on [0,1] interval and prefers a party that is closest to one's own position
  - 2 ways to determine winner: 1. Deterministic 2. Random
- What does the winner care about?
  a) Party cares about being elected
  b) Party cares about position of the elected party

Model of deterministic voting

- Voters: not strategic – just vote for a candidate with a platform closest to their own position; assume infinitely many voters, as if they are spread evenly across the whole interval [0,1]
- Winning party:
  - Votes for party that is closer to own position
  - As one voter is infinitely small compared to all voters, it does not matter how an indifferent voter votes.
- Ties & summary:
  - We assume that parties have probability 0.5 of winning each
    → if the parties decided to locate at a=b and
    → if $\frac{a+b}{2} = 1 - \frac{a+b}{2}$, that is, each party gets equal number of votes
  - Assume a<b. Note that:

$$\underbrace{\frac{a+b}{2}}_{\text{Share of votes for } A} > \underbrace{1 - \frac{a+b}{2}}_{\text{Share of votes for } B} \quad \text{implies } a > 1 - b.$$

  - Summary:

$$\text{The winner is party} \begin{cases} A & \text{if } a > 1 - b \\ B & \text{if } a < 1 - b \\ A \text{ or } B & \text{if } a = 1 - b \text{ or } a = b \end{cases}$$

Model of probabilistic voting

- Winning party:
  - The larger the share of voters who usually support a given party, *the more likely it is* to win
  - Specifically, we assume:

    If parties are positioned at a and b, with $a < b$, party A wins with probability $\frac{a+b}{2}$; party B wins with probability $\left(1 - \frac{a+b}{2}\right)$.
    If parties are positioned at $a = b$, parties with with probability 1/2 each.
    If parties are positioned at a and b, with $a > b$, party A wins with probability $\left(1 - \frac{a+b}{2}\right)$; party B wins with probability $\frac{a+b}{2}$.

Topic 10: Bilateral Trade and Myerson-Satterthwaite impossibility result

Information constraint

- Asymmetric information: agents do not have the same information; imposes constraints
- Constraints can take many forms:
  - Physical: e.g. bundle outside her budget set
  - Informational constraints

Informal set-up

- Buyer (B) & Seller (S)
  1. If B values object more than S, trade is efficient, object should change hands
  2. If S values object more than B, trade is inefficient, object should not change hands
- If it is not known with certainty if trade is efficient, it may be impossible for B and S to agree on efficient trades
- "Impossible": no auction, no bargaining, and no other mechanism

Formal set-up

- A seller, with item to sell:
  - May have low valuation $V_S = 0$ or high valuation $V_S = 0.9$ with equal probabilities
- A buyer, interested to buy item:
  - May have medium valuation $V_B = 0.1$ or high valuation $= 1$ with equal probabilities
- When is trade efficient? $V_S < V_B$: $(0,0.1), (0,1)$
- When is trade not efficient? $V_S > V_B$: (0.9,0.1)
- We want to ensure all efficient trades happen

Mechanism 1: seller makes offer

BUYER'S DECISION:

- Backward induction: if $V_B \geq p$, buyer accepts, otherwise rejects
- Seller's offer:
  - $p < 0.1$
  - $\mathbf{p = 0.1}$

  In both cases, $v_B = 0.1$ and $v_B = 1$ will accept. $p < 0.1$ is worse than $p = 0.1$ for a seller.

  - $0.1 < p < 1$
  - $\mathbf{p = 1}$

  In both cases, only $v_B = 1$ will accept. $0.1 < p < 1$ is worse than $p = 1$ for a seller.

SELLER'S DECISION:

- Which price is better, p=0.1 or p=1?