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The excess return on a security is its return over the risk-free rate. The CAP implies that the excess return of security *I* is proportional to its systematic risk (*beta*)

$$E[r_{i.t}] - r_f = \beta_i \times Market\ Risk\ Premium$$

This implies that all securities have the same excess return to risk ratio so:

$$\frac{E[r_{i.t}] - r_f}{\beta_i} = Market\ Risk\ Premium = \frac{E[r_{j.t}] - r_f}{\beta_j}$$

This implies a relationship that can be used to estimate beta. This is shown by the relationship previously:

$$E[r_{i.t}] - r_f = \beta_i \times Market\ Risk\ Premium$$

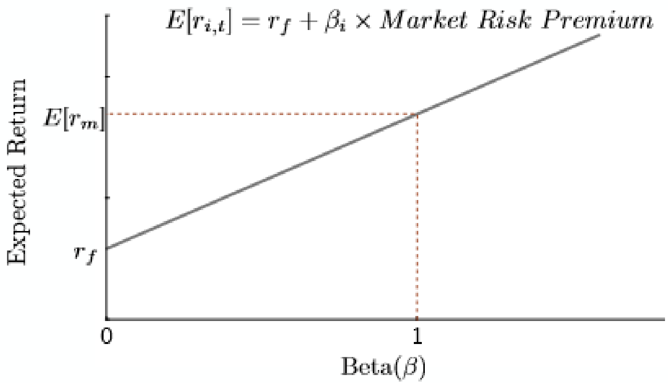
$$E[r_{i.t}] - r_f = \beta_i \times (E[r_{m.t}] - r_f)$$

This implies we can use observed data to find beta: which is done by a linear regression line

$$r_{i,j} - r_f = \beta_i \times (E[r_{m.t}] - r_f)$$

Security Market Line

The SML plots the expected return on a security as a function of its beta



i. Securities or portfolios with betas less than 1 are expected to return less than the market
ii. Securities or portfolios with betas greater than 1 are expected to return more than the market
iii. A securities' expected return is not a function of it total risk (standard deviation) but depends on its systematic risk (beta)

Combing the two equations that satisfy the portfolio's expected returns and the Capital Asset Pricing Model:

$$R_P = w_A \times R_A + w_B \times R_B + \cdots$$

$$E(R_P) = w_A \times E(R_A) + w_B \times E(R_B) + \cdots$$

$$E(R_P) = w_A \times \left(r_f + \beta_A \times MRP\right) + w_A \times \left(r_f + \beta_A \times MRP\right) + \cdots$$

$$E(R_P) = \left(w_A r_f + w_B r_f + \cdots\right) + (w_A \beta_A + w_B \beta_B + \cdots) \times MRP$$

$$E(R_P) = r_f + +(w_A \beta_A + w_B \beta_B + \cdots) \times MRP$$

Since the weights must add up to 1, then

$$\beta_P = \omega_A \times \beta_A + \omega_B \times \beta_B + \cdots$$

Project Specific Discount Rates

A firm that uses an average discount rate instead of project-specific discount rates will:

- Not invest in positive NPV low risk projects
- Over invest in negative NPV high risk projects

Because of these investment decisions, firms using average discount rates may be riskier over time.

Weighted Average Cost of Capital (WACC)

The analysis of risk and return shows that the expected return premium on any asset is proportional to is systematic risk (*beta*)

So, firms trying to attract investment must:

- Offer investments that meet the return expectations of the markets. A project must have a positive NPV when discounted at the appropriate expected return.
- Providing returns is a necessary cost to companies to gain investor capital

Costs of Capital is the required rate of return a company must offer investors for a project to compensate them for risk. Consequently, it is the discount rate a company should use when valuing projects.

Portfolios and Discount Rates

The *project is like a portfolio* where investments into different securities will yield a weighted average return on these securities, which is the expected return of the project

The analysis of risk and returns shows that the return on a portfolio is a weighted average of the returns of the securities in the portfolio. Consequently, a *single project* firm trying to attract investment must:

- Offer a return equal to the weighted average of the expected returns of the firm's securities
- Ensure the weights are based on market values as these are the values used by investors

The *weighted average* of a project's cost of capital for each securities used in financing. Weights are the fractional amounts of each security's total project market value

$$r_{WACC} = w_E \times \begin{pmatrix} Cost\ of \\ Ordinary\ Shares \end{pmatrix} + w_P \begin{pmatrix} Cost\ of \\ Preference\ Shares \end{pmatrix} + w_D \begin{pmatrix} Cost\ of \\ Debt \end{pmatrix}$$

Levered and Unlettered Firms

A firm without debt is called an unlevered firm, while a firm with debt outstanding is called a levered firm. The weighted average cost of capital for an unlevered firm is just the cost of equity.