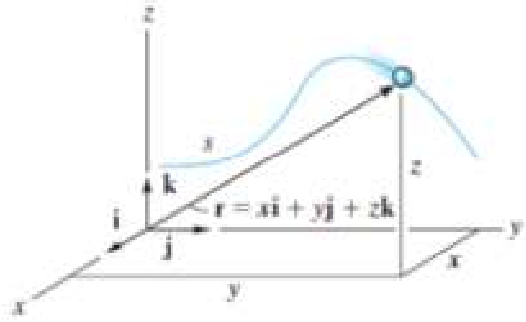
TRC2201 - Mechanics Summary Notes

Contents

Cartesian Coordinates



Position:

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

Velocity:

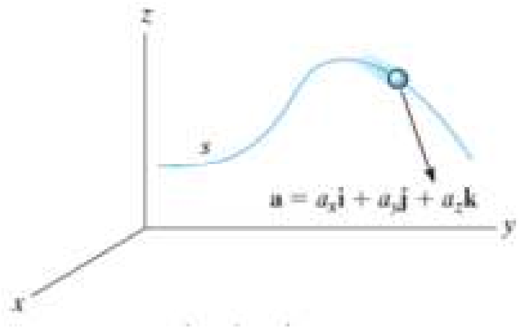
$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

$$= \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$

Velocity Magnitude:

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Acceleration:



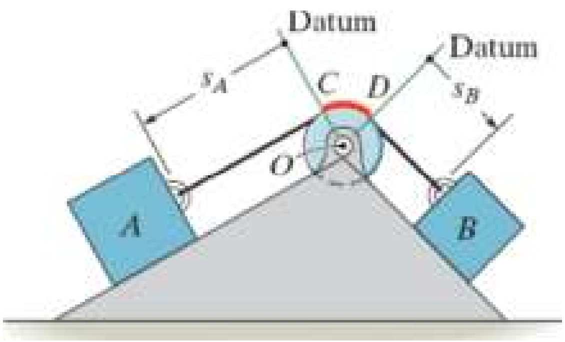
$$a_x = \dot{v}_x = \ddot{x}$$

$$a_y = \dot{v}_y = \ddot{y}$$

$$a_z = \dot{v}_z = \ddot{z}$$

Dependent Motion

Motion of one particle can depend on motion of another (e.g. if connected by cord)

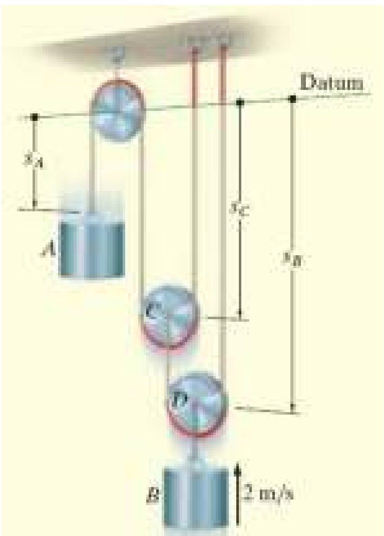


$$s_A + l_{CD} + s_B = l_T$$

Constants

$$\frac{d}{dt}(s_A + l_{CD} + s_B) = \frac{dl_T}{dt}$$

$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \quad \Rightarrow \quad v_B = -v_A$$



$$s_A + 2s_C = l_1$$

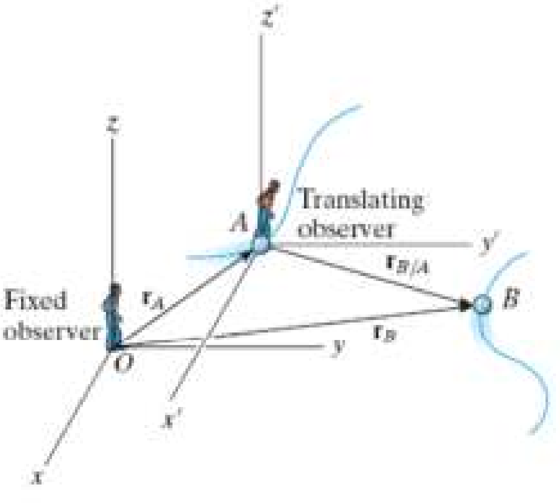
$$s_B + (s_B - s_C) = l_2$$

$$s_A + 4s_B = 2l_2 + l_1$$

$$\frac{d}{dt}(s_A + 4s_B) = \frac{d}{dt}(2l_2 + l_1) = 0$$

$$v_A + 4v_B = 0$$

Relative Motion of Two Particles



(1) Fixed reference xyz

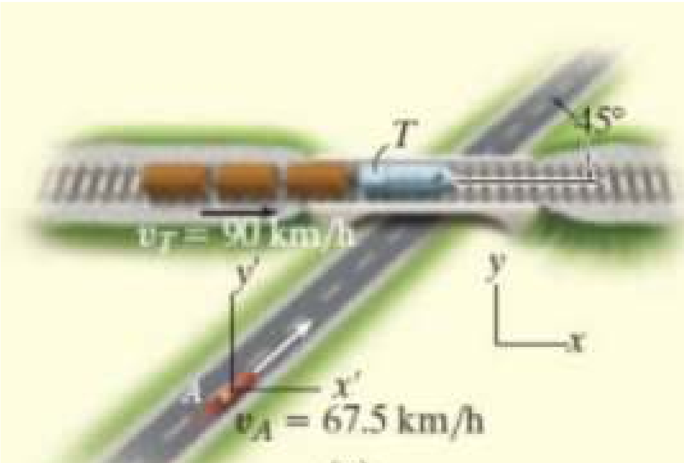
(2) *Translating* reference $x'y'z'$ attached to particle A

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

v_A and v_B are **absolute** velocity measured from fixed frame.
$V_{B/A}$ is **relative** velocity observed from translating reference.
Each vector represented in **i** and **j** components.

Example



(a)

$$\boldsymbol{v_T} = \boldsymbol{v_A} + \boldsymbol{v_{T/A}}$$

$$\boldsymbol{v_T} = 90\boldsymbol{i}$$

$$\boldsymbol{v_A} = 67.5\cos 45\,\boldsymbol{i} + 67.5\sin 45\,\boldsymbol{i}$$

$$\boldsymbol{v_{T/A}} = 42.3\boldsymbol{i} - 47.7\boldsymbol{j}$$

$$|v_{T/A}| = 63.8\ km/h$$

$$\tan\theta = \frac{(v_{T/A})_y}{(v_{T/A})_x} = \frac{47.7}{42.3}$$

$$\theta = 48.40^\circ$$

Force and Acceleration

Law of Gravitational Attraction

$$F = G\frac{m_1 m_2}{r^2}$$

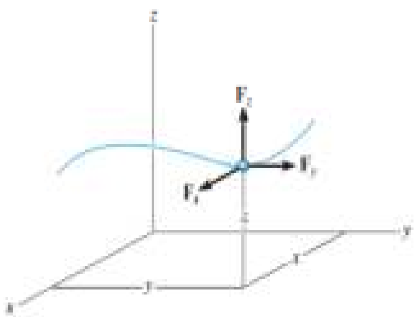
F = force of attraction between particles

G = universal constant of gravitation

m_1, m_2 = mass of each of the particles

Equations of Motion in Rectangular Coordinates

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum F_z = ma_z$$

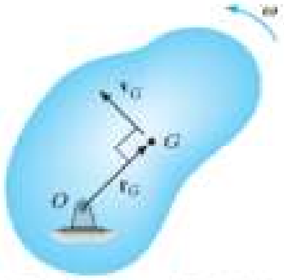


Work and Energy

Total kinetic energy of rigid body:

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

translational rotational



Rotation About a Fixed Axis

For rotation about point O:

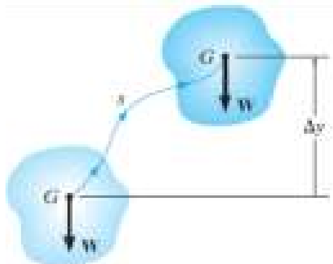
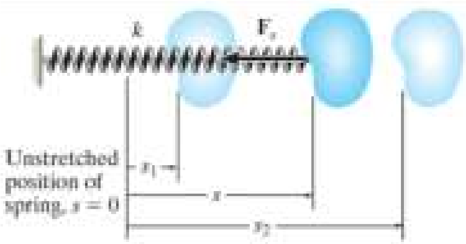
$$v_G = r_G \omega$$

$$so\ T = \frac{1}{2}\omega^2 (mr^2 + I_G)$$

$$\therefore T = \frac{1}{2} I_O \omega^2$$

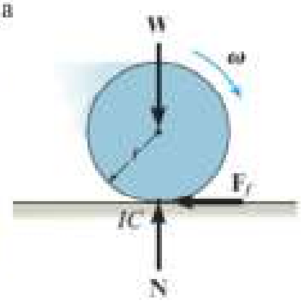
Work of Forces

Work done **on** object/spring is **negative**.

Weight Force	Spring Force
	
$U_W = -W \Delta y$	$U_s = -\left(\frac{1}{2} k s_2^2 - \frac{1}{2} k s_1^2\right)$

Forces that do NOT do work:

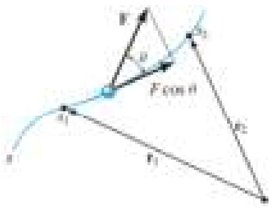
- Reactions at a pin support about which a rigid body rotate.
- Normal reaction acting on a body that moves along a fixed surface.
- Gravity force when the center of mass moves horizontally.
- *Friction force when slipping does not occur.*

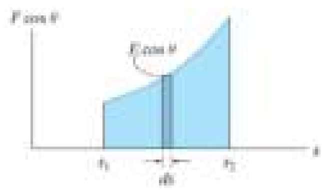


Work of moment:

$$U_M = \int_{\theta_1}^{\theta_2} M \, d\theta$$

Work of a Variable Force:





$$U_{1-2} = \int_{r_1}^{r_2} F.dr = \int_{s_1}^{s_2} F \cos\theta \, ds$$

Conservation of Energy

$$T_1 + V_1 + \left(\sum U_{1-2}\right)_{noncons} = T_2 + V_2$$

kinetic From external force potential