

TRC2201 - Mechanics Summary Notes

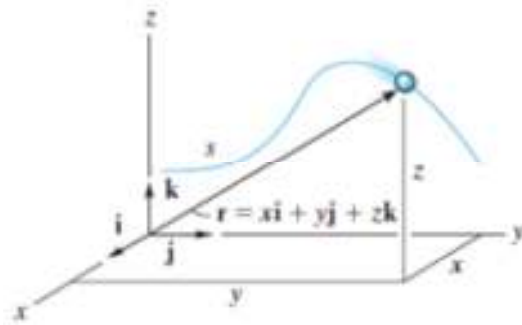
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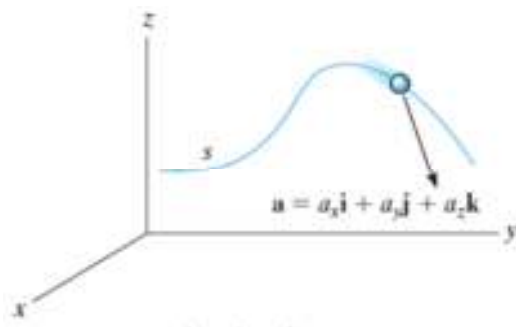
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Cartesian Coordinates



Acceleration:



Position:

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

Velocity:

$$\begin{aligned}\vec{v} &= v_x\vec{i} + v_y\vec{j} + v_z\vec{k} \\ &= \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}\end{aligned}$$

Velocity Magnitude:

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

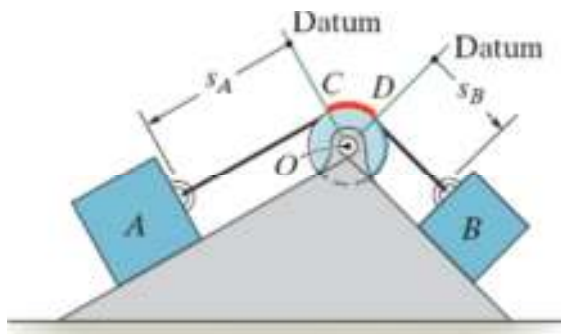
$$a_x = \dot{v}_x = \ddot{x}$$

$$a_y = \dot{v}_y = \ddot{y}$$

$$a_z = \dot{v}_z = \ddot{z}$$

Dependent Motion

Motion of one particle can depend on motion of another (e.g. if connected by cord)

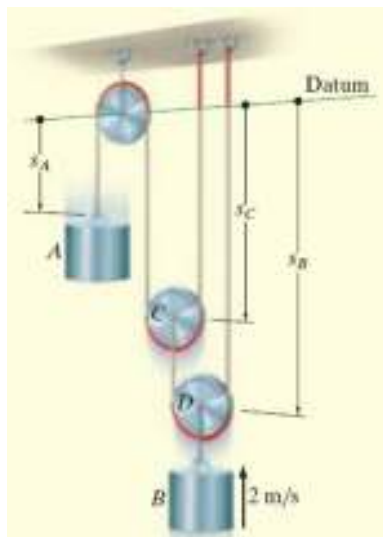


$$s_A + l_{CD} + s_B = l_T$$

Constants

$$\frac{d}{dt}(s_A + l_{CD} + s_B) = \frac{dl_T}{dt}$$

$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \Rightarrow v_B = -v_A$$



$$s_A + 2s_C = l_1$$

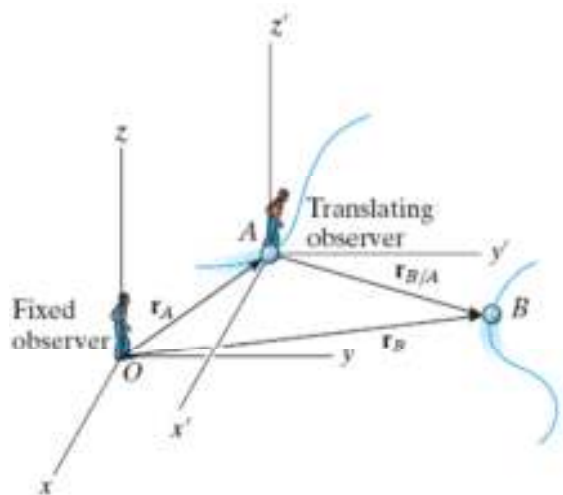
$$s_B + (s_B - s_C) = l_2$$

$$s_A + 4s_B = 2l_2 + l_1$$

$$\frac{d}{dt}(s_A + 4s_B) = \frac{d}{dt}(2l_2 + l_1) = 0$$

$$v_A + 4v_B = 0$$

Relative Motion of Two Particles



(1) Fixed reference xyz

(2) Translating reference $x'y'z'$ attached to particle A

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

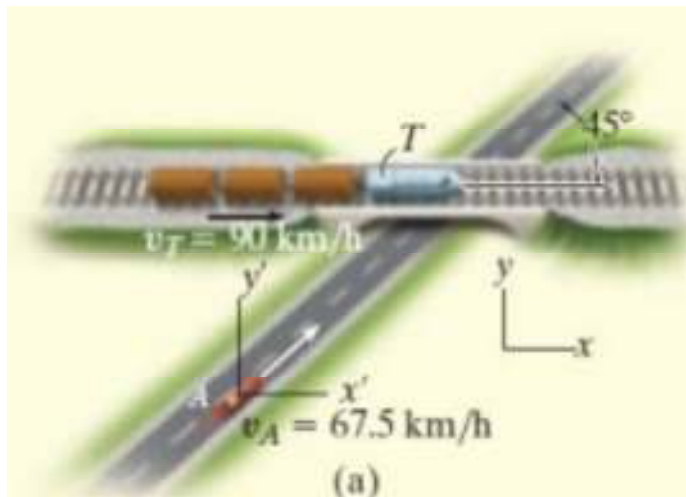
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

v_A and v_B are **absolute** velocity measured from fixed frame.

$v_{B/A}$ is **relative** velocity observed from translating reference.

Each vector represented in i and j components.

Example



$$\vec{v}_T = \vec{v}_A + \vec{v}_{T/A}$$

$$\vec{v}_T = 90\mathbf{i}$$

$$\vec{v}_A = 67.5 \cos 45^\circ \mathbf{i} + 67.5 \sin 45^\circ \mathbf{j}$$

$$\vec{v}_{T/A} = 42.3\mathbf{i} - 47.7\mathbf{j}$$

$$|\vec{v}_{T/A}| = 63.8 \text{ km/h}$$

$$\tan \theta = \frac{(v_{T/A})_y}{(v_{T/A})_x} = \frac{47.7}{42.3}$$

$$\theta = 48.40^\circ$$

Force and Acceleration

Law of Gravitational Attraction

$$F = G \frac{m_1 m_2}{r^2}$$

F = force of attraction between particles

G = universal constant of gravitation

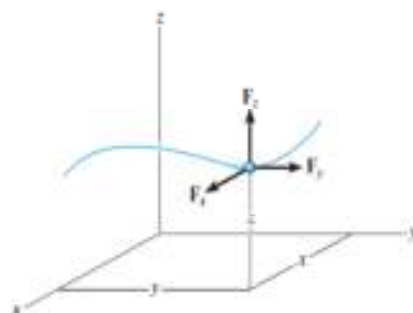
m_1, m_2 = mass of each of the particles

Equations of Motion in Rectangular Coordinates

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$



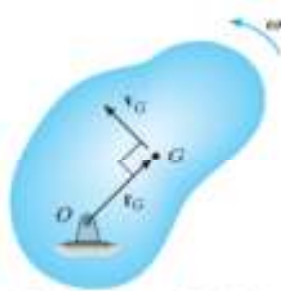
Work and Energy

Total kinetic energy of rigid body:

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

translational

rotational



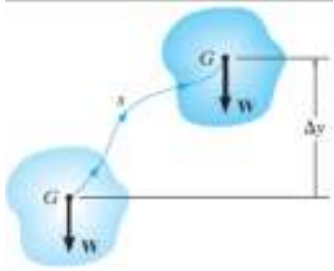
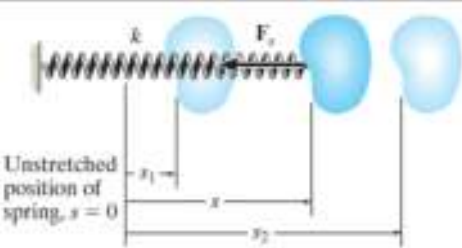
Rotation About a Fixed Axis

For rotation about point O:

$$\begin{aligned} v_G &= r_G \omega \\ \text{so } T &= \frac{1}{2} \omega^2 (m r_G^2 + I_G) \\ \therefore T &= \frac{1}{2} I_O \omega^2 \end{aligned}$$

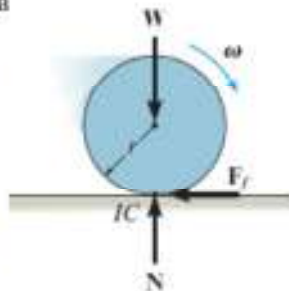
Work of Forces

Work done **on** object/spring is **negative**.

Weight Force	Spring Force
	
$U_W = -W \Delta y$	$U_s = -\left(\frac{1}{2} k s_2^2 - \frac{1}{2} k s_1^2 \right)$

Forces that do NOT do work:

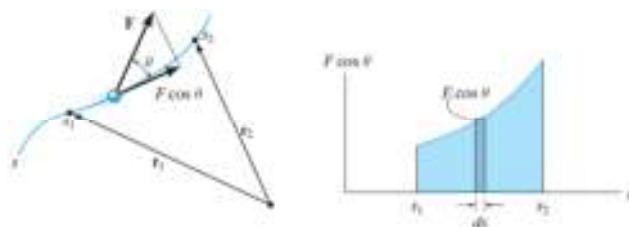
- Reactions at a pin support about which a rigid body rotate.
- Normal reaction acting on a body that moves along a fixed surface.
- Gravity force when the center of mass moves horizontally.
- Friction force when slipping does not occur.



Work of moment:

$$U_M = \int_{\theta_1}^{\theta_2} M d\theta$$

Work of a Variable Force:



$$U_{1-2} = \int_{r_1}^{r_2} F \cdot dr = \int_{s_1}^{s_2} F \cos \theta ds$$

Conservation of Energy

$$T_1 + V_1 + \left(\sum U_{1-2} \right)_{\text{noncons}} = T_2 + V_2$$

kinetic

From external force

potential