

**FNCE 10002**

# **PRINCIPLES OF FINANCE NOTES**

**Week 1-12**

## Week 3 – Applications in Financial Mathematics I

Annual loan payment is an ordinary annuity.

In a loan amortization schedule:

- Interest paid = (previous period's principal) x interest rate
- Principal repaid = Loan payment – Interest paid
- Principal balance remaining = Previous period's principal – Principal repaid
- Principal balance = PV of remaining loan payments

Effective annual interest rate is the annualized rate that takes account of compounding within the year.

$$r_e = \left(1 + \frac{r}{k}\right)^k - 1$$

where  $r$  = annual percentage rate

$k$  = compounding frequency (1 for annual, 2 for semi-annual, 12 for monthly, etc)

$r/k$  is the per period interest rate

Properties of the Effective Annual Interest Rate:

- $r = r_e$  only when compound frequency is one year ( $k=1$ ); otherwise  $r_e$  will always exceed  $r$ .
- The effective annual rate with continuous compounding (compounding frequency with  $k$  approaching infinity) is  $r_e = e^r - 1$ .

The more frequent you compound (the bigger the  $k$ ), the higher the effective annual interest rate you get.

Short term debt instruments:

- Matures within the year – typically in 90 and 180 days
- Issuer has the contractual obligation to make promised payment at maturity

### Long term debt instruments:

- Matures after several years
- May or may not promise a regular interest (or coupon) payment
- Issuer has the contractual obligation to make all promised payments.

Face (or par) value is the dollar amount paid at maturity.

- Usually \$1000 or its multiples unless otherwise mentioned

Coupon (or interest) rate is the interest rate promised by the issuer.

- It is expressed as a percentage of the face value.

Coupon (or interest) payment is the periodic payment made to bondholders.

- Coupon payment (C) = Coupon rate x Face Value

The price of discounted securities is computed as the present value of the face value at a market determined yield to maturity (YTM).

Yield to maturity is the rate of return earned by an investor who holds the security until it matures assuming no default occurs on the security.

$$\text{Price of a discounted security} = \frac{\text{Face value}}{[1 + (\frac{n}{365}) \times r_d]}$$

- If YTM falls, price will increase
- If YTM rises, price will fall

$$\text{Price of a coupon paying bond} = \left(\frac{C_1}{r_d}\right) \left(1 - \frac{1}{(1 + r_d)^n}\right) + \left(\frac{\text{Face Value}}{(1 + r_d)^n}\right)$$

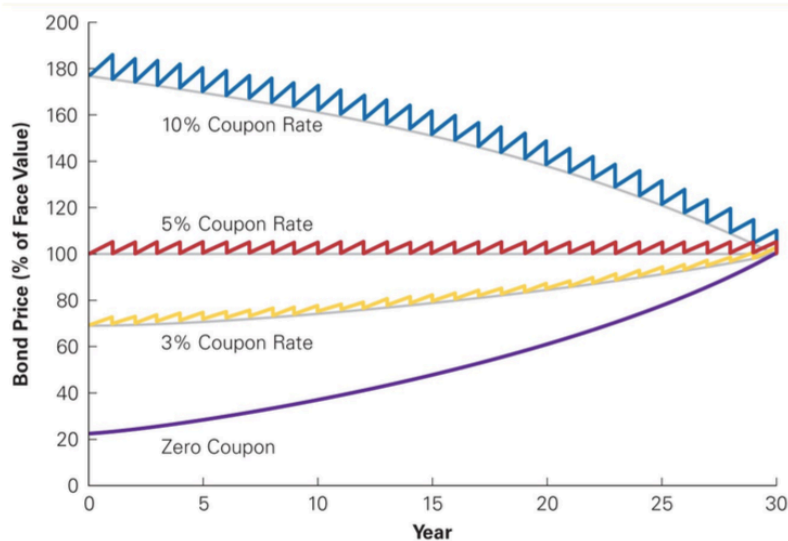
The **yield to maturity** is also the **interest rate** that **discounts** the bond's future cash flows to equal the **market price today**.

- It is also called the **internal rate of return**.

**Relationship between Coupon Rates and Yields to Maturity:**

- When **Price = Face Value**
  - The bond is selling at **par**
  - **YTM = Coupon Rate**
- When **Price > Face Value**
  - The bond is selling at a **premium**
  - **YTM < Coupon rate**
- When **Price < Face Value**
  - The bond is selling at a **discount**
  - **YTM > Coupon rate**

### *Bond Prices Over Time*



Source: Berk and DeMarzo, Figure 6.1. The bond's yield to maturity is assumed to be 5%.

The prices of longer maturity bonds are more sensitive to changes in interest rates than the prices of shorter maturity bonds.

- This is because investors have to bear a higher interest rate risk with longer maturity bonds.

Bonds with higher coupon rates are less sensitive to changes in the yield to maturity changes than no-coupon paying bonds.

SAMPLE

## Week 4 – Applications in Financial Mathematics II

Ordinary shares represent ownership in the issuing company's cash flows (earnings and dividends).

- It is first issued and sold in the primary market – typically via an initial public offering
- Once listed, it is traded on secondary markets – such as the ASX, NYSE
- It provides an infinite stream of earnings and dividends
- It has no maturity date

Why dividends and not earnings?

- Shareholders care about the cash flow they receive, not what a firm earns.
- A firm's earnings are typically not all paid out as dividends, so we should consider dividends, and not earnings, when valuing shares.

The General Dividend Discount Model

$$P_0 = \sum_{n=1}^{\infty} \frac{D_n}{(1+r_E)^n}$$

The Constant Dividend Growth Model

- Dividends grow at a constant growth rate (g) forever

$$P_n = \frac{D_{n+1}}{r_E - g}$$

*Note: n+1, not n!*

The link between earnings and dividends:

$$D_n = \alpha E_n$$

where  $\alpha$  is the dividend payout ratio – proportion of earnings paid out as dividends.

- If payout ratio doesn't change over time, the growth rate in dividends will be equal to the growth rate in earnings.

Assuming the number of shares outstanding is constant, a firm can **increase its dividends** by:

- Increasing its earnings
- Increasing its dividends payout ratio

What can a firm do with their **earnings**:

- Pay them out to shareholders as dividends
- Retain the earnings and reinvest them

Any **increase in future earnings** will result from **new investments made** from **earnings retained** by the firm.

- Change in earnings = New investment x Return on new investment
- Earnings: New Investment / Retention rate
- Growth = Retention rate x Return on new investment

There is an implication that **lowering dividend** to **increase investment** will **raise the stock price**, **if, and only if**, the new investments have a **positive NPV**.

**Calculating Share Price** at  $t=0$ :

1. Obtain dividends up to where  $g$  becomes constant
2. Obtain  $P_n$  (after which dividends growth is constant)
3. Add the **present values of dividends** and the **present value of the price** at time to get  $P_0$

$$P_0 = \sum_{t=1}^n \left( \frac{D_t}{(1+r_E)^t} \right) + \frac{P_n}{(1+r_E)^n}$$

**Preference shares** are shares which give their holders **preference over ordinary shareholders** regarding the **payment of dividends and repayment of capital** in case of liquidation.

- *Price of plain vanilla preference share* ( $P_0$ ) =  $\frac{D_p}{r_p}$

The **P/E ratio** is the ratio of the current market price to expected (or current earnings) per share.

- Expected (forward) P/E ratio =  $\frac{P_0}{E_1}$
- Current (trailing) P/E ratio =  $\frac{P_0}{E_0}$

The **expected P/E ratio** is the amount investors are willing to pay now for \$1 of future expected earnings.

Expected and current P/E ratios

$$\frac{P_0}{E_1} = \frac{\alpha}{r_E - g} \quad \text{and} \quad \frac{P_0}{E_0} = \frac{\alpha(1+g)}{r_E - g}$$

This implies as **expected P/E ratio rises**:

- The payout ratio ( $\alpha$ ) rises
- The growth rate of dividends ( $g$ ) rises
- The required return on equity ( $r_E$ ) falls

If a firm's **earnings and dividends do not grow** ( $g=0$ ) and it **pays out all earnings** ( $\alpha = 1$ ) as **dividends**, the price is  $P_0 = \frac{E_1}{r_E}$ .



## WEEK 5 – Modern Portfolio Theory and Asset Pricing I

### Ordinary Shares Returns

- ❖  $R_t = (P_t + D_t - P_{t-1})/P_{t-1}$
- ❖  $R_t = D_t/P_{t-1} + (P_t - P_{t-1})/P_{t-1}$
- ❖  $R_t = \text{Dividend yield} + \text{Percent price change}$

### Bond Returns

- ❖  $R_t = (P_t + C_t - P_{t-1})/P_{t-1}$
- ❖  $R_t = C_t/P_{t-1} + (P_t - P_{t-1})/P_{t-1}$
- ❖  $R_t = \text{Coupon yield} + \text{Percent price change}$

**Arithmetic average return** measures the return earned from a single, one period investment over a specific time horizon.

$$\bar{R} = (R_1 + R_2 + \dots + R_T)/T$$

**Geometric average return** measure the return earned per period from an investment over an investor's entire time horizon.

$$\bar{R}_g = [(1 + R_1)(1 + R_2) \dots (1 + R_T)]^{1/T} - 1$$

If **returns are constant** (example: returns are always 10%), **geometric return = average return**.

The **observed (realized) risk** of a security is measured by the **variability** in its realized returns around the arithmetic average return.

The realized variance of returns is...

$$Var(R) = \frac{1}{T-1} \sum_{t=1}^T (R_t - \bar{R})^2$$

The realized standard deviation of returns is...

$$SD(R) = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (R_t - \bar{R})^2}$$

**Least to Most Risky Investments:**

- Treasury bills
- Corporate bonds
- World Portfolio
- Standard & Poor's 500
- Small stocks

The **expected return** is the **expected outcome** measured as the **weighted average** of the **individual outcomes**.

$$E(r) = p_1 R_1 + p_2 R_2 + \dots + p_n R_n$$

The **variance** or **standard deviation of returns** is the **measure of dispersion** around the **expected return**.

- The **greater** the **dispersion**, the **higher** the **uncertainty** and **risk**.
- Variance and standard deviation take into account **returns above and below** the **expected return**.

Investors are typically concerned with returns **below** the **expected return** (**downside risk**).

$$Var(r) \text{ or } \sigma^2 = p_1 [R_1 - E(r)]^2 + p_2 [R_2 - E(r)]^2 + \dots + p_n [R_n - E(r)]^2$$

$$SD(r) = \sigma$$

Investors are assumed to be **risk averse**.

- The **higher** the **variance or standard deviation of returns** the **worse** off the investor
- Their objective is to **minimize the risk of portfolio of investments** given a **desired level of expected return**; or to **maximize the expected return of portfolio** of investments given a **desired level of risk**.

**Portfolio risk falls** as the **number of securities in the portfolio increases**.

- However, portfolio risk **can't be entirely eliminated**
- The **risk that can't be eliminated** is called **systematic risk**

A portfolio's **expected return** is the **weighted average** of the component **securities' expected returns**.

- **Weights** are **percentages** of the investor's original **wealth invested in each security**.
- It is assumed **all funds** are **invested in two securities**.

$$E(r_p) = x_1 E(r_1) + x_2 E(r_2)$$

- ❖  $x_j$  = Amount invested in security  $j$  / Total amount invested
- ❖ **Note:**  $x_1 + x_2 = 1$  and  $x_1 = 1 - x_2$  (or  $x_2 = 1 - x_1$ )

A portfolio's **variance** is the **weighted average** of the **variance of its component securities** and the **covariance between the securities' returns**.

$$\text{Var}(r_p) \text{ or } \sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \sigma_{12}$$

- ❖  $\sigma_{12}$  or  $\text{Cov}(r_1, r_2)$  = Covariance between securities 1 and 2

- The **first** and **second term** can **only** be **positive**
- The **last term** can be **negative, positive, or zero**

$$\text{SD}(r_p) \text{ or } \sigma_p = [x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \sigma_{12}]^{1/2}$$

The *covariance of returns* [ $\sigma_{12}$  or  $\text{Cov}(r_1, r_2)$ ] measures the level of *comovement* between security returns

$$\sigma_{12} = p_1[r_{11} - E(r_1)][r_{21} - E(r_2)] + \dots + p_n[r_{1n} - E(r_1)][r_{2n} - E(r_2)]$$

❖  $r_{jk}$  = Return on security  $j = 1, 2$  in state  $k = 1, 2, \dots, n$

Positive covariance ( $\sigma_{12} > 0$ )

- Above (below) average returns on security 1 tend to coincide with above (below) average returns on security 2.

Negative covariance ( $\sigma_{12} < 0$ )

- Above (below) average returns on security 1 tend to coincide with below (above) average returns on security 2.

Covariance = 0 ( $\sigma_{12} = 0$ )

- Security 1's return tends to move independently of security 2's return.
- i.e. security 2 is a risk-free security.

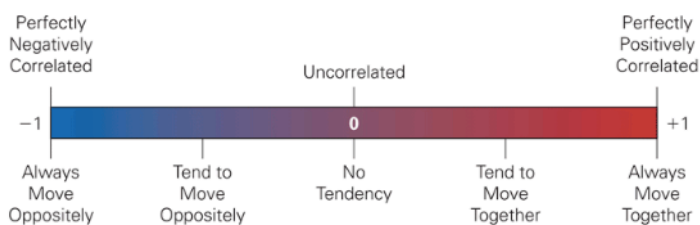
## Correlation Between Security Returns

❖ The *correlation of returns*,  $\rho_{12}$  or  $\text{Corr}(r_1, r_2)$ , is a “standardized” measure of comovement between two securities

$$\rho_{12} = \sigma_{12} / \sigma_1 \sigma_2$$

❖ **Note 1:** The sign of the return correlation is the *same* as the sign of the return covariance

❖ **Note 2:**  $-1 \leq \rho_{12} \leq +1$



❖ The covariance of returns can be rewritten as...

$$\sigma_{12} = \sigma_1 \sigma_2 \rho_{12}$$

## Correlation Between Security Returns

- ❖ There are *two* definitions for a portfolio's variance (and standard deviation) of returns
- ❖ Using the *covariance* of returns...
  - ❖  $\text{Var}(r_p) \text{ or } \sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1x_2\sigma_{12}$
- ❖ Using the *correlation* of returns...
  - ❖  $\text{Var}(r_p) \text{ or } \sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1x_2\sigma_1\sigma_2\rho_{12}$

The *risk-return trade-off* in a *two-security portfolio* depends on the *level of co-movement (or correlation)* between their *returns*.

- *Return correlation of +1* (perfect positive correlation)
  - $\rho_{12} = +1$
  - There are **no gains** from *diversification* in this case as the portfolio's risk (standard deviation) is a weighted-average of the risks (standard deviations) of the two securities
- *Return correlation of -1* (perfect negative correlation)
  - $\rho_{12} = -1$
  - There will be **maximum gains** from *diversification*
  - It is possible to construct a *zero-risk portfolio*
- *Return correlation between +1 and -1* (general case)
  - $-1 < \rho_{12} < +1$
  - **Some diversification benefits** always exist
  - $\sigma_p^2 = x_1^2 \sigma_1^2 + (1 - x_1)^2 \sigma_2^2 + 2x_1(1 - x_1)\sigma_1\sigma_2\rho_{12}$
  - The *diversification benefit* depends on the *correlation of returns*;
    - The *lower* the *correlation*, the *higher* the *diversification benefits*

In the case where there is a **perfect negative correlation** and the **minimum variance** (or **standard deviation**) of the portfolio is 0 ( $\sigma_p = 0$ ), the standard deviation expression is simplified into:

$$\text{➤ } 0 = \sigma_p = x_1(\sigma_1) - (1 - x_1)(\sigma_2)$$

**Portfolio leveraging** is the strategy where an investor **borrow funds** at the **risk-free rate of return** and **invests** all the **available funds** in a **risky security** (or portfolio).

- The **more** we **borrow**, the **riskier** the investment gets.

**Short selling** refers to **borrowing shares**, and selling them now with a contractual promise to **buy them back** later at (an **expected**) **lower** price.

- This method of leveraging **increases** portfolio risk!
- Borrowing and short selling security A and investing proceeds in security B implies  $x_A < 0\%$  and  $x_B > 100\%$  such that  $x_A + x_B = 1$ .

**Portfolio leveraging** and **short selling** **increases** the **expected return** of a portfolio and the **risk** associated with that portfolio!